

SCIENCE OF TSUNAMI HAZARDS

Journal of Tsunami Society International

Volume 39

Number 4

2020

EVALUATION OF THE IMPACT OF MAJOR EARTHQUAKES ON EXCITING LONG PERIOD FREE-EARTH LITHOSPHERIC OSCILLATIONS, ATMOSPHERIC-IONOSPHERIC PERTURBATIONS, AND FAR-FIELD TSUNAMI-LIKE WATER LEVEL FLUCTUATIONS

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ABSTRACT

The whole Earth resonates like a bell with normal modes of resonance at distinct frequencies. When extremely large earthquakes strike, the Earth's free oscillations are excited. These excited, long period, enhanced earth oscillations have frequencies which have a tendency to resonate over long periods of time after a major earthquake. The Great Sumatra-Andaman Islands Earthquake of 26 December 2004 - the largest event in the last half century – was the first event in the Moment Magnitude (Mw 9) category to be recorded with modern digital instruments. The earthquake generated distinct stronger free oscillations of the Earth's lithosphere. Also, further coupling of these oscillations reportedly resulted in distinct atmospheric as well as ionospheric perturbations of certain modalities and frequencies. The present paper examines whether the excited stronger "spheroidal normal modes" of free earth oscillations could have contributed as well to tsunami generation enhancement, and to the lasting and persistent tidal oscillations that were recorded in the Andaman Sea and elsewhere. The present review further examines the efficiency of coupling of excited stronger solid free earth oscillations with the ocean, and analyzes whether these could have contributed significantly to the destructiveness of the tsunami that was observed in the Indian Ocean, or to unusual far-field water level fluctuations recorded by tide gauges in the Pacific and Atlantic Oceans – which cannot be supported by calculated tsunami travel times. Additionally to the 2004 event, other major earthquakes, volcanic and meteorological events are similarly examined as to their possible excitation of free Earth oscillations or coupling with the sea surface and the atmosphere, to generate far-field, tsunami-like sea level fluctuations or meteotsunamis.

Keywords: *Surface seismic waves; free earth oscillations; Tsunamis, Earthquake source observations; 26 December 2004 earthquake; spheroidal, Toroidal modes; Rayleigh & Love Wave Interactions*

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1. INTRODUCTION

The Earth is not a perfect sphere. Beneath and above its surface, the distribution of rock and mineral formations is uneven, thus causing pockets of varying density and therefore of gravity. Also, near the equator, the earth's spinning rotation creates a bulge, thus raising its surface and resulting in a decrease in the pull of gravity, while near the north and southern poles the gravitational acceleration is highest. Also, in certain regions where tectonic plates move apart, the earth's crust is not as thick and gravity is weaker. Finally, large surface features on the earth's surface such as mountain ranges increase the force of gravity. NASA satellites probing and measuring the Earth's gravity when they approach regions of higher density are pulled forward, at slightly higher speed and slow down somewhat when they reach regions of lesser crustal density.

Furthermore, Planet Earth can be considered to be a mechanical system with a finite body of mass. All mechanical systems possess natural vibrations that can be excited by internal or external forces. Researchers at UC Santa Barbara and Tokyo Institute of Technology have determined that the Earth vibrates continuously and suspect atmospheric turbulence as the cause of tiny spheroidal waves (Tanimoto et.al., 1990; Tanimoto, 2001). However, when a large earthquake occurs, different seismic phases are generated which propagate away from the source region, through and around the earth. Typical periods for compressional P- and shear S-waves are of the order of a few seconds, while the surface seismic waves have longer periods.

Often, these seismic phases interfere both constructively and destructively with each other in a resonant way, so that their arrival and graphic and digital signatures at different seismic stations may vary significantly. Additionally, when large earthquakes strike, the Earth's free oscillations are excited - thus the whole Earth resonates like a bell with normal modes of resonance at distinct frequencies. These self-excited, long period, free earth oscillations have frequencies which have a tendency to resonate over long periods of time after a major earthquake. To understand how the earth's natural vibrations are being excited and enhanced by earthquakes, we need to review some of the progressive historical developments that have taken place in the field of seismology.

The history of studies relating to seismic waves goes back to Poisson in 1829 (Poisson, 1829) and Lamb in 1882 (Lamb, 1982). Studies of seismic surface waves begun with Rayleigh in 1885 and with Love in 1911 (Love, 1911). Such early studies helped determine the Earth's upper-mantle structure and rheological heterogeneity. Most of our subsequent understanding about the Earth's interior has come from the application of the ray-theoretic methods to seismological data. Ray theory is useful for periods shorter than a minute. Travel-times of seismic phases constitute an important component of the ray theory concept. However, before reviewing the enhancement of the long period free-earth oscillations caused by large earthquakes and their coupling with the atmosphere and ionosphere, or with the seas and oceans in increasing near-field tsunami heights or far-field tsunami-like wave activity observations or measurements, we need to also review

briefly the earlier pioneering research in seismology that has further increased our understanding of the earth's near surface and internal structure (Pararas-Carayannis, 2000a, 2000b; 2000c). The following section pertains to a brief review of such early studies and of research in seismology that led to the understanding of tectonic interactions and crustal displacements and perturbations responsible for tsunami generation mechanisms, as well to far-field tsunami-like sea level fluctuations that cannot be supported by normal wave refraction and tsunami travel times.

Also, the present study is a preliminary evaluation on whether great earthquakes such as the 26 December 2004 Sumatra Earthquake, or other events of large magnitude, affect the earth's free oscillations, and whether they contribute to the enhancement of near-field generation of the destructive tsunami waves that were observed in the Indian Ocean and adjacent seas, or to the unusual far-field water level fluctuations recorded by tide gauges in both the Pacific and the Atlantic Oceans, which do not conform with tsunami travel times based on wave refraction and ocean bathymetry. In addition, the present study provides a brief review of the Earth's free modes of natural oscillations and of their excitation by major or great tsunamigenic earthquakes.

2. EARLY HISTORY OF PIONEERING RESEARCH IN SEISMOLOGY

Although earthquakes from the beginning plagued humanity and millions of lives were lost, the causes of earthquakes were not studied systematically until the 19th Century. A book addressing seismic hazards was written by an Irish Engineer, Robert Mallet and was entitled, "The Great Neapolitan Earthquake of 1857". The first book on "Principles of Observational Seismology", which resulted from his investigation of this particular earthquake, was a milestone in the evolution of seismology.

The subsequent history of evolution of seismology is quite interesting. Mallet and his contemporary Englishman, John Milne, were the pioneers of such research (Mair, 2013). However Milne is considered to be the father of seismology because he made a remarkable impact on the study of earthquakes by designing and constructing earthquake measuring devices and by collecting and publishing earthquake data and maps of worldwide earthquake distribution. This early data formed the basis for the initial understanding of earthquakes and for measuring important seismic parameters.

The early history of pioneering research in seismology is extensively documented in the scientific literature and in a summary of a previous issue of "Science of Tsunami Hazards" (Pararas-Carayannis, 2000c). This report mentions that the first seismographs in the United States were installed in 1887 at the Berkeley campus of the University of California and at the Lick Observatory at Mount Hamilton, California. Prior to the great San Francisco earthquake of 1906, earthquake research in U.S.A. had advanced very slowly compared to efforts in Japan and Europe. However, around the turn of the century, a small number of U.S. Geological Survey scientists and geology professors at a few U.S. universities, began to contribute earthquake data and to compile lists of historic earthquakes in the U.S. and around the world. At that time, and in spite of the earlier work in Japan and Europe, still very little was known about earthquakes, how and where they occurred, or the risks they presented. The modern theory of plate tectonics had not yet been proposed and was still

years away. Nothing was known about the Earth's free oscillations or their excitement by earthquakes, or the mechanisms of tsunami generation, in general.

The great 1906 San Francisco earthquake was a major event that triggered the interest of many scientists and resulted in numerous scientific investigations. Comprehensive studies of this earthquake and of the San Andreas fault-system in California marked the beginning of modern seismology in the U.S.A. and the understanding of the earth's internal structure. According to the historical record, the exhaustive investigation and surveys of the 1906 San Francisco earthquake illustrated the importance of collecting valid, extensive, and repetitive data on earthquakes, their effects, and on the faults on which they occur. These comprehensive studies of this particular earthquake - formed the basis for the understanding of earthquakes in California and elsewhere in the USA and around the world.

A final report on the great San Francisco earthquake - often referred to as the Lawson report - was published in 1908. The report was a comprehensive compilation of detailed studies by more than twenty scientists on the 1906 earthquake's damage, intensities, the slip movement along the San Andreas Fault, the seismograph records of the earthquake from around the world, and the underlying geology in northern California. One of the scientists was Henry Reid, a professor of Geology at Johns Hopkins University. Professor Reid examined extensively the ground displacements of the 1906 earthquake on the San Andreas fault. Based on his observations he reached the conclusion that this earthquake must have involved stored energy and accumulated stress, which was suddenly released, thus he proposed the "elastic rebound theory" of earthquakes. His concept of elasticity continues to influence today's scientific thinking about earthquakes (Pararas-Carayannis, 2000a).

3. EARLY STUDIES OF THE EARTH'S SEISMICITY - INTERACTIONS OF SURFACE SEISMIC WAVES WITH THE LITHOSPHERE, HYDROSPHERE, ATMOSPHERE, IONOSPHERE AND ENHANCEMENT OF FREE-EARTH OSCILLATIONS.

As stated above, the earlier studies of the earth's seismicity were initiated by S.D. Poisson (Poisson, 1829), by Mallet and Milne in 1857 (the two founding fathers of engineering seismology)(Mair, 2013), by Horace Lamb (Lamb, 1882) on vibrations of an elastic sphere (Ewing et.al 1957; Feynman, 1963), by Rayleigh in 1885 on seismic waves propagated along the plane surface of an elastic solid known as "Rayleigh waves", and by A.E.H. Love (Love, 1911 a,b), who developed a mathematical model of surface seismic waves, known as "Love Waves". Both types of surface seismic waves known as "Rayleigh" and "Love" can be damaging to structures, but particularly the "Love" waves that result in horizontal accelerations. However, none of these earlier studies correlated surface or deeper seismic motions of earthquakes as contributing to the earth's free oscillations.

To help understand the excitation of free Earth oscillations - which are standing waves - mainly by the surface seismic waves or by their combined coupling with the sea surface and the atmosphere, to generate near-field or far-field, tsunami-like sea level fluctuations or meteotsunamis, the following section provides a brief review of their interactions of both "Rayleigh" and "Love" waves. These interactions may be further influenced by sea tides, and "spring tides", depending on the moon's vernal and di-urnal position and on its gravitational alignment with other celestial bodies.

3.1 “Rayleigh” Wave Interactions

The “Rayleigh” waves are known to cause both vertical and horizontal ground motions. These waves are more pronounced near the earth’s surface and as they propagate, they lift and drop the ground and also move it horizontally. Their stress occurs mainly on the vertical plane (Fig. 1). Because of the vertical motions and coupling with the lithosphere, the hydrosphere, the atmosphere and the ionosphere, the Rayleigh waves can result in oscillations in these regimes and may have an exciting effect on the free earth oscillations and may be responsible in somewhat enhancing the observed far-field tsunami-like waves – the travel times of which cannot be supported by tsunami propagation based on the shallow water wave-equations. The following is a brief description of the motions on the surface of the earth caused by “Rayleigh” wave (Fig 1).

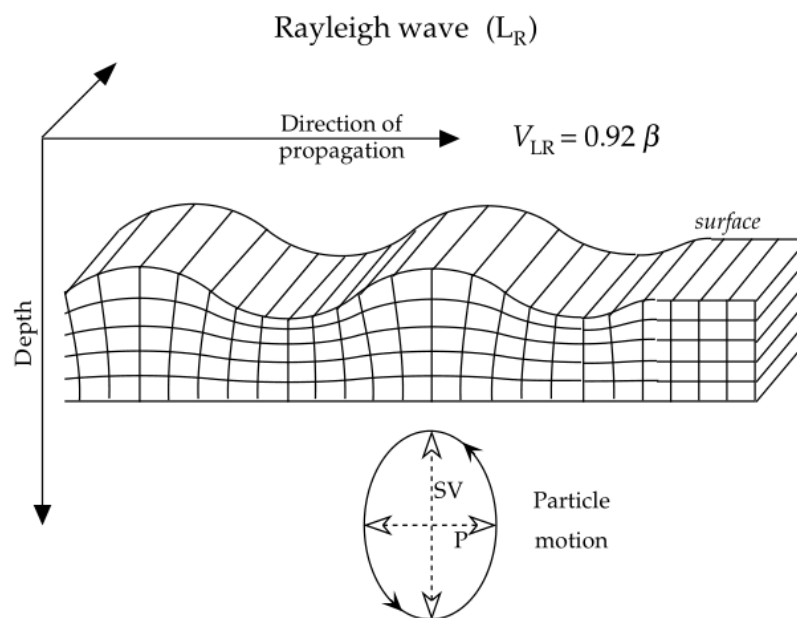


Fig. 1. Diagram of “Rayleigh” wave propagation on the surface of the earth. The resulting motions include both longitudinal and traverse motions that decrease exponentially as the distance from the surface increases. The waves are more pronounced near the surface and their stress occurs mainly on a vertical plane.

Since Rayleigh waves from large earthquakes involve vertical motions, research results on delayed ionospheric oscillations of Rayleigh waves from large earthquakes, could be detected and recorded by high-frequency Doppler sounding techniques (Furumoto, 1970). To illustrate such coupling with both the atmosphere and the ionosphere, this particular study used the 10 MHz recording of Rayleigh waves to estimate the initial phase of the source of the Kuril earthquake of 11 August 1969. From such recordings also of ionospheric perturbations, the initial motion from this quake appeared to be downward and, because of the rapidity of such recording, this approach to tsunami source mechanism

estimation was considered to be a useful indicator of potential tsunamigenesis, and thus was recommended for use by the Pacific Tsunami Warning System (PTWC) (Pararas- Carayannis, 2000 a, b, c).

3.2 “Love” Wave Interactions

In contrast, the “Love” waves travel without vertical displacement and they move the ground from side to side in a horizontal plane, but at right angles to the direction of propagation. “Love” waves are known to be particularly damaging to the foundations of structures because of the horizontal ground motions they generate and the horizontal shearing of the ground (Fig. 2). The waves are more pronounced near the surface and their stress occurs mainly on a horizontal plane. The resulting motions include lateral ground motions and also decrease exponentially as the distance from the surface increases. Although the “Love” waves do not contribute directly to tsunami wave heights, the horizontal stresses and particle motions they cause may result in bookshelf type of failures of sedimentary structures near a tsunami generating area and thus contribute to the enhancement of tsunami heights. Such was the case with the great 2011 Tohoku-Oki earthquake in Japan (Pararas-Carayannis, 2011).

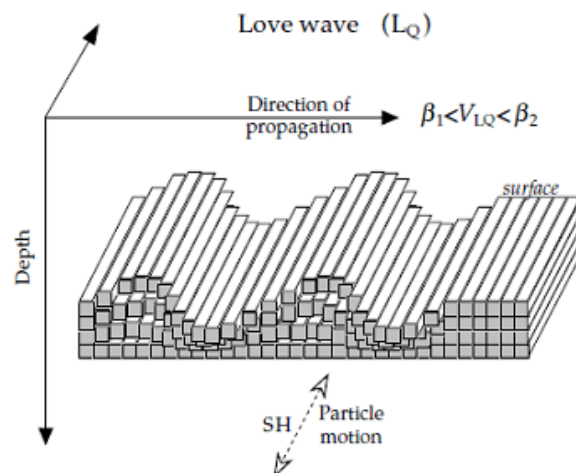


Fig. 2. Diagram of “Love” wave propagation on the surface of the earth.

3.3 Modes of the Earth’s Free Oscillations

Large earthquakes generate two distinct types of free modes of natural oscillations (standing waves) in the Earth – the spheroidal which are equivalent to the Rayleigh Waves, and the toroidal which are equivalent to the Love waves. The spheroidal mode, has two sub-classifications and the toroidal has three. Both types of oscillations have an infinite number of modes (Ben-Menahem, 1964; Brune, 1964; Buland, 1964,1981; Dahlen, 1968, Dahlen & Sailor, 1979; Dahlen & Tromp, 1998; Geller & Stein, 1979). Free oscillations have relatively long periods of hours to days. In the present study, we refer only to the earth’s free oscillations modes of only certain frequencies that are excited by major or great earthquakes.

4. BASIC MOTIONS OF THE EARTH'S FREE OSCILLATION MODES

The influence of gravity on the vibrations of an elastic globe were studied as early as 1898 (Bromwich, 1898) and by many more researchers in later years (Dahlen & Smith, 1975; Woodhouse & Dahlen, 1978; Valette, 1986; Woodhouse, 2008). The normal free oscillations of the Earth were extensively studied by numerous other researchers following the 1964 Chilean Earthquake, the largest tsunamigenic event in recorded history (Brune, 1964; Ben-Menahem, 1964; Dahlen, 1968; Buland, 1981; Park Et Al, 2005; He & Tromp, 1996; Lognonné & Clévéde, 2002).

As described in the scientific literature, the basic motions of the Earth's free oscillations can be illustrated with the following diagram of the spheroidal (radial and tangential (football)) and toroidal modes (Fig. 3).

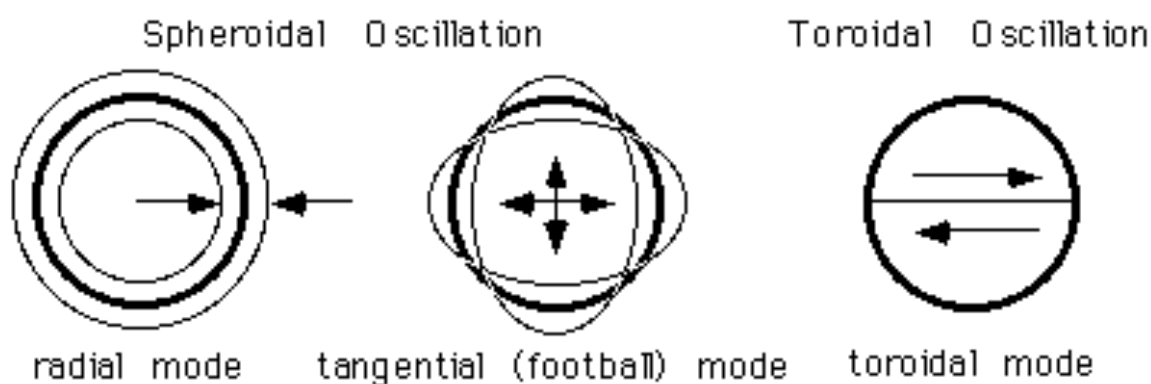


Fig. 3. Spheroidal and Toroidal Free Earth Oscillations

4.1 Long-period Toroidal oscillations (T)

The toroidal oscillations involve shear motions parallel to the surface of the earth and are not as significant as the spheroidal modes. Relative displacement motion for toroidal oscillations is always perpendicular to the radius vector. These oscillations involve only the earth's crust and mantle. They are equivalent to the Love waves.

4.2 Long-period Spheroidal oscillations (S)

The displacement for spheroidal oscillations has both radial and tangential components. They are equivalent to the Rayleigh waves. The most important of the Earth's "spheroidal normal mode" is the so called "football" or "rugby" mode, abbreviated as OS2. It has a 53.8 minute period or 0.31 mHz frequency. Another spheroidal normal mode of motion - also with a radial component - is the "balloon" or "breathing" mode, abbreviated as OS0. It has a 20.6 minute period or 0.81 mHz frequency. The third of the spheroidal modes is the OS3 with a period of 35.5 minutes or 0.47 mHz frequency.

4.3 Tiny Natural Earth Vibrations Generation by Atmospheric Turbulence

For the purpose of determining the Earth's natural oscillation modes, scientists from the University of California at Santa Barbara and from the Tokyo Institute of Technology, analyzed gravimeter data from 1983 to 1994 and found 61 days to be seismically "quiet". For this period of time, they identified several such natural modes in the range of 2 to 7 milli-Hz – which were very small vibrations with periods of hundreds of seconds. The acceleration of material in the solid Earth which were produced by these spheroidal waves was only in the order of nano-gals, or 10^{-9} cm/sec² thus suspecting that the cause of the tiny vibrations resulted by atmospheric turbulence. (Tanimoto et al., Geophysical Research Lett., May 15; toshiro@magic.geol.ucsb.edu)

4.4 Impact of Major Earthquakes on Long Period Free-Earth Oscillations on the Lithosphere

Subsequent studies of the normal-mode theory on the free Earth's oscillations are based on the geometrical ray theory of optics (Singh & Rani 2011). This particular approach views these oscillations as standing waves rather than traveling waves, and their peaks are identifiable in the amplitude spectrum of a seismogram.

Theoretically, the free oscillations from an arbitrary earthquake source can be derived by solving the governing radial differential equations of motion and correlating the earth's elastic-gravitational response, which can be expressed as a sum of such free oscillations or normal modes. The same methodology can be used to calculate synthetic seismograms on a spherical earth by normal mode summation, and interpret the propagation of Love and Rayleigh waves and mode-ray duality. However, such a theoretical solution requires many assumptions as to the Earth's sphericity, rotation and rheological homogeneity. Since the Earth is neither homogeneous nor completely spherical, the calculated modes will differ from what actually occurs when a large earthquake strikes – as discussed in the following section.

5. EXCITMENT BY EARTHQUAKES OF THE EARTH'S FREE OSCILLATIONS

Planet Earth vibrates continuously even in the absence of earthquake activity. The Earth's free oscillations were observed for the first time in the early 1960's. The systematic study of the earth's free oscillations begun after the Great Chilean earthquake of 22 May 1960 - the largest during the 20th Century. Since then, the earth's free oscillations and their excitment by earthquakes have been extensively studied by many researchers. As mentioned, such studies determined two distinct types of free modes of natural oscillations – the spheroidal, which are equivalent to the Rayleigh waves, and the toroidal, which are equivalent to the Love waves. The spheroidal mode has two sub-classifications and the toroidal has three. Both types of free cossillations have an infinite number of modes. This occurs because the Earth is spherically asymmetric and has lateral heterogeneities within it. Furthermore, the influence of its rotation splits the modes. However, at the very low frequency range (below 10 mHz), which has been used extensively in the last 40 to 50

years, the free oscillations can be observed and provide valuable information on the whole Earth (Montagner & Roult, 2004).

The earth's free oscillations have relatively long periods of hours to days. These modes are excited and altered by earthquakes. As mentioned, normal modes have been used to calculate accurate synthetic seismograms and derive the centroid moment tensor of earthquakes. Also, the relative excitement of ultra-long period spheroidal oscillations has been used to calculate more accurately the energy and moment magnitude of great earthquakes. For example, the Kamchatka Earthquake of 1952 had a 57 min fundamental mode. The following section pertains to recent major or great earthquakes. The relative excitement of certain modes of spheroidal oscillations were used by some researchers in calculating more accurately the energy release and moment magnitudes of some of the recent strong earthquakes – particularly of the 9 June 1994 in Bolivia and of the 26 December 2004 in Indonesia.

5.1 The 9 June 1994 Earthquake in Bolivia

The 9 June 1994 earthquake in Bolivia was unique in the sense that it was unusually strong, had a focal depth of 636 kilometers and a rapid rupture rate which ranged from 1 to 3 km/second (Silver *et al.*, 1995).

Based on analysis of IRIS broad-band seismograms for this earthquake and by inversion of the sections with duration of 330 seconds which included several phases of refracted compressional P-waves and Rayleigh as well as Love waves – the latter with duration of long period (175 to 250 seconds) - the dip-slip mechanism for this quake and its scalar moments were determined, both agreeing and corresponding to a moment magnitude $M_w=8.3$ (Kikuchi & Kanamori, 1994).

5.2 The 26 December 2004 Earthquake

The Great Sumatra-Andaman Islands Earthquake of 26 December 2004 - the largest event in a last half century - was the first event in the Moment Magnitude (M_w 9) category to be recorded with modern digital instruments. This earthquake generated distinct strong free oscillations of the Earth's lithosphere. Also, further coupling of these oscillations resulted in atmospheric as well as ionospheric perturbations of certain modalities and frequencies.

This earthquake was extensively studied and reviewed in the scientific literature (Montagner & Roult, 2004; Kayal & Wald, 2004; Stein and Ocal, 2005; Pararas-Carayannis, 2005; Gower, 2005). Based on the history of past earthquake events and a clearly identifiable seismic gap along western Sumatra (see Fig. 4 below), the occurrence of this great earthquake was predicted as early as 1989 and included in a report to the United Nations Development Program (UNDP) (Pararas-Carayannis, 1989 a; 1989 b).

With the sponsorship and coordination of UNESCO-Intergovernmental Oceanographic Commission, UNDP sponsored and funded a field investigation and subsequently approved the proposed plan (Pararas-Carayannis, 1989a). This report recommended

specifically a five year plan of action designed to mitigate the impact of the pending disaster with the installation of monitoring instrumentation and the establishment of a Regional Tsunami Warning System that could provide timely warnings for the countries bordering the Indian Ocean.

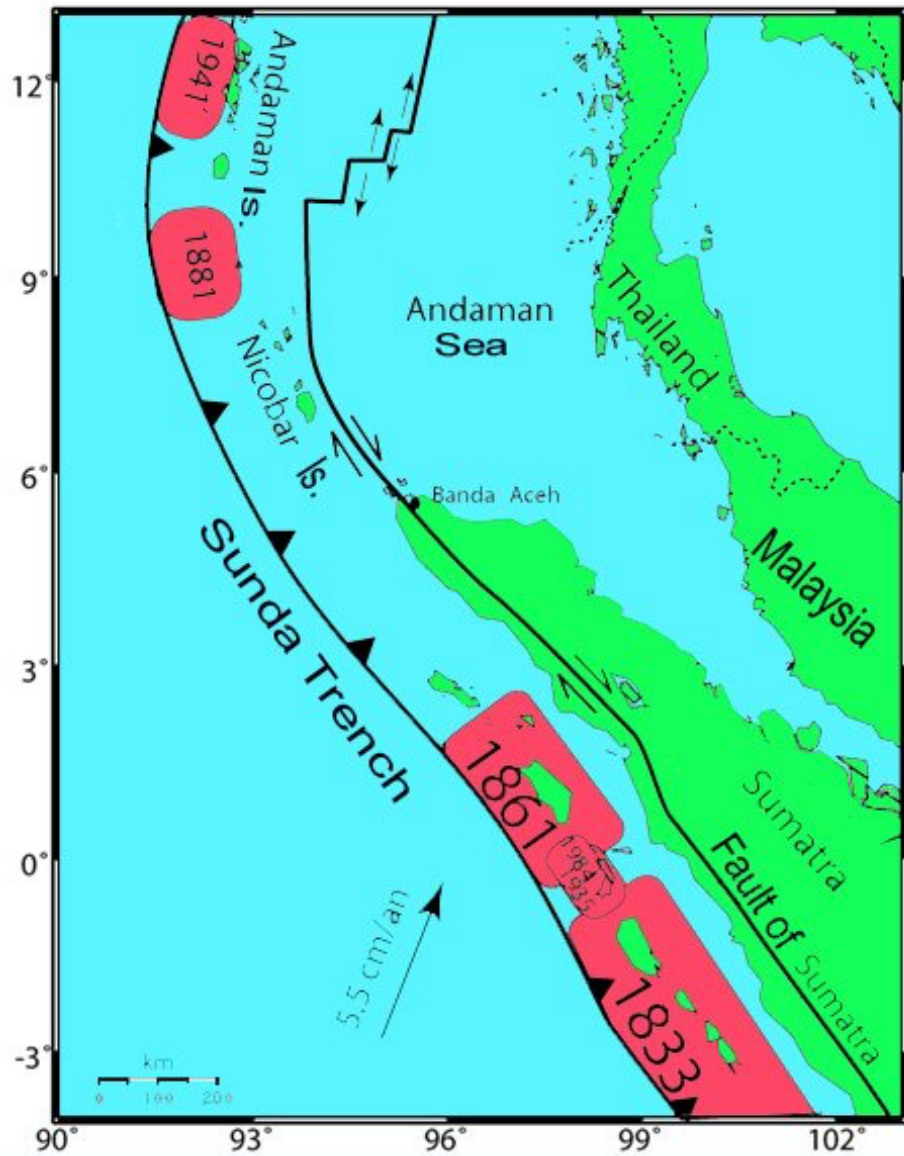


Fig. 4. Identifiable seismic gap along western Sumatra
After <http://www.drgeorgepc.com/Tsunami2004Indonesia.html>

The anticipated great earthquake finally occurred a few years later on 26 December 2004 along the indicated seismic gap as shown in Figure 5, but included an extension to the Nicobar and Andaman Islands, the segments that ruptured by earthquakes in 1881 and 1941.

5.2a Spheroidal and Toroidal Modes of the 26 December 2004 Earthquake

From a technical point of view, this earthquake provided high-quality seismic data which was recorded by the broad-band stations of the Federation of Digital Seismograph Networks (FDSN). These recordings made it possible to observe a very large collection of split modes, not only of the spheroidal but also of the toroidal modes (Montagner & Roullet, 2004; Stein and Okal, 2005; <http://www.iris.iris.edu/sumatra/>).

M9.0 Andaman - Nicobar Islands Earthquake of 26 December 2004

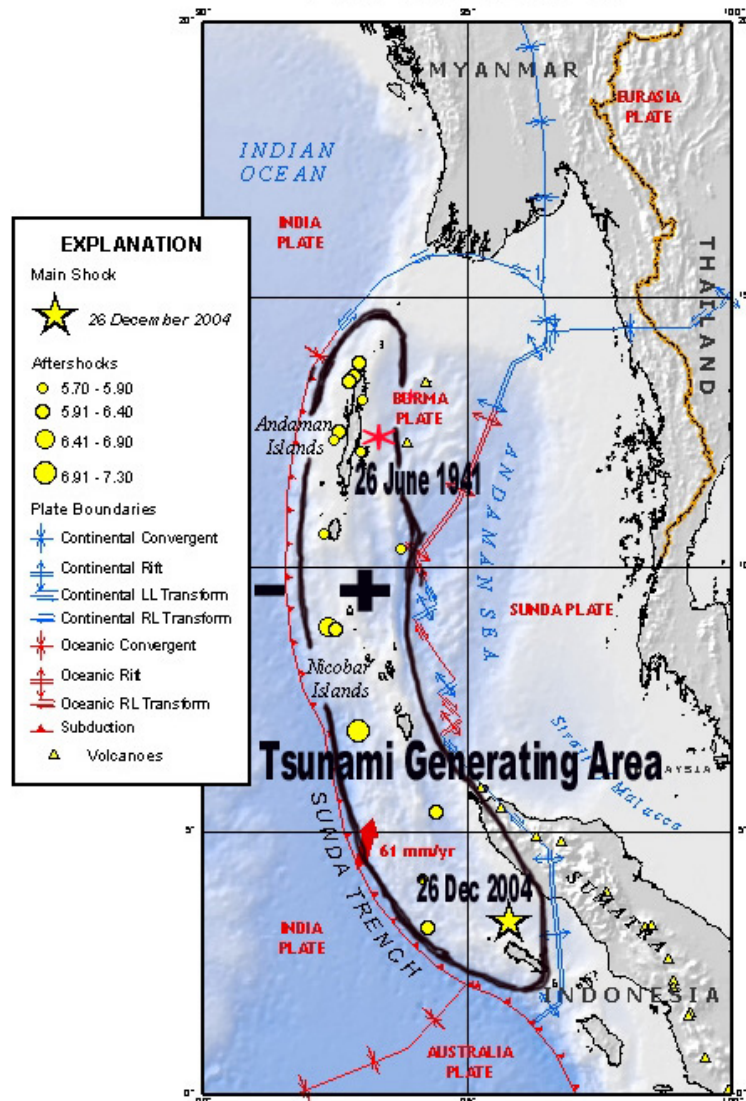


Fig. 5 Tsunami Generating Area of the 26 December 2004 Earthquake (After <http://www.drgeorgepc.com/Tsunami2004Indonesia.html> - Modified USGS map showing the earthquake epicenter, the distribution of initial major aftershocks, and the interaction of major tectonic plates along the Sunda Trench)

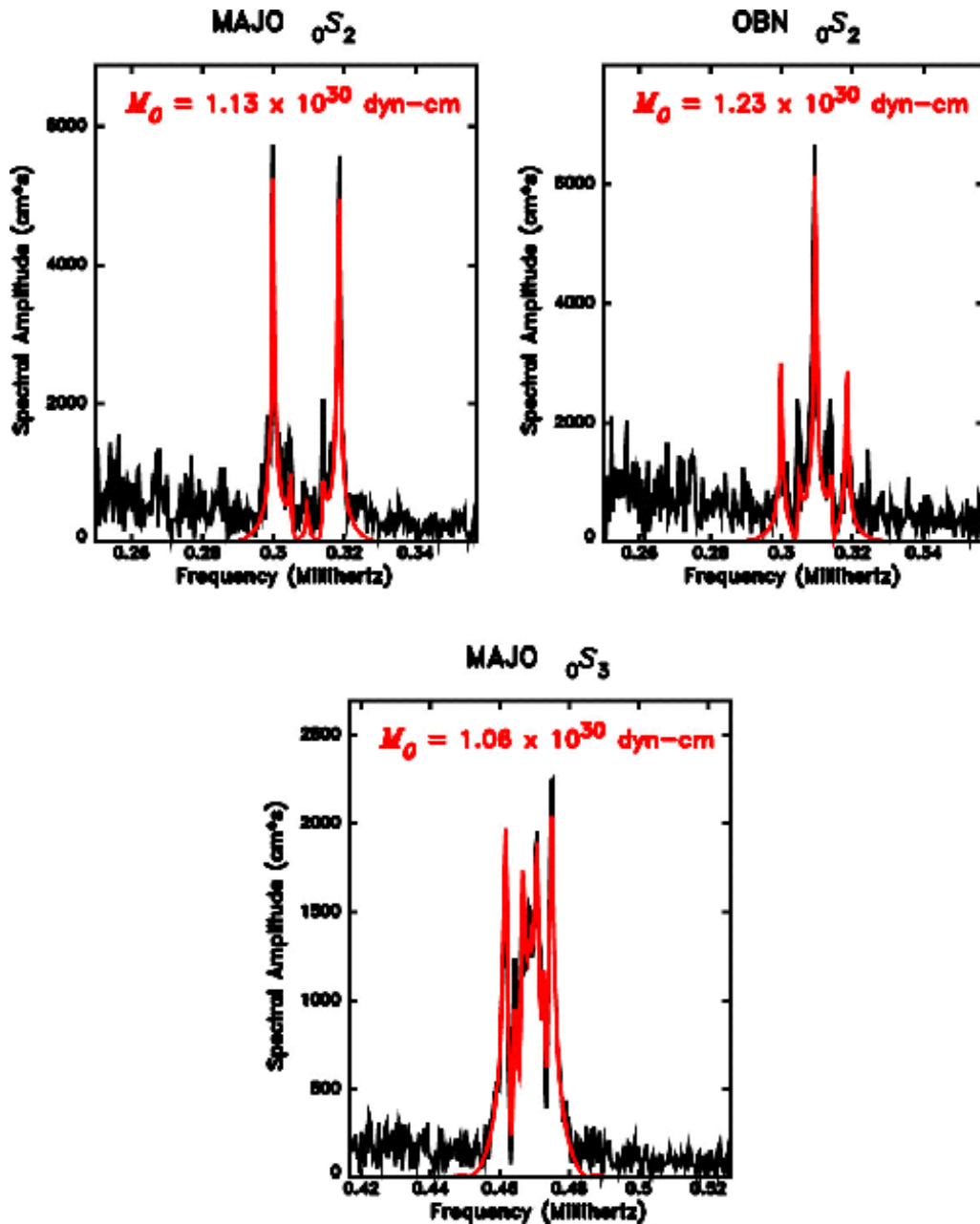


Fig. 6. Analysis of the longest period normal modes of the earth, OS2 and OS3, of the 26 December 2004 earthquake, yielded a greater moment magnitude $M_w = 9.3$ rather than the $M_w = 9.0$ that was initially measured from long period surface waves (source: <http://www.iris.iris.edu/sumatra/> Credit: Seth Stein and Emile Okal, Department of Geological Sciences, Northwestern University)

Another study used the relative excitement of ultra-long period spheroidal oscillations to calculate more accurately the energy and moment magnitude of this event. In fact, analysis of the longest period normal modes of the earth – the OS2 and OS3 – were used to

calculate its energy release and moment magnitude. Based on Fourier analysis of long seismograms and further analysis of normal mode multiplets of the 0S2 and 0S3 spheroidals with periods of about 3,231 seconds (53.85 min.) and 2,134 seconds (35.5667 min), the Moment of the December 26, 2004 Sumatra earthquake was determined to be 1.3×10^{30} dyn-cm. (Stein and Okal, 2005). This was approximately three times larger than the 4×10^{29} dyn-cm that had been measured from long period surface waves (see Fig. 6 above). Analogous re-evaluations of other major or great historical earthquakes can be expected to result in greater estimates of Mw magnitudes, than those conventionally calculated and reported.

Hence, the 2004 earthquake's ultra-long period magnitude, was re-evaluated to be Mw = 9.3, which was significantly greater than the previously estimated Moment Magnitude Mw = 9.0. – making this earthquake the second largest ever instrumentally recorded.

Even with the initially lower Moment Magnitude of MW = 9.0 estimate, the scale of motion was particularly unique and remarkable for seismic recordings. In terms of units of displacement, the peak-to-peak ground motions observed globally were greater than one centimeter for the long oscillations (100+sec) of the Love (G) and Rayleigh (R) surface waves.

5.3 The 11 March 2011 Earthquake in Japan

The great Tohoku-Oki earthquake (Mw 9.0) of 11 March 2011 off the Pacific coast of Honshu Island in Japan was a megathrust event which produced displacement of at least 40 m.. was extremely destructive and generated a very destructive and anomalously high tsunami. Prior tsunamigenic earthquakes in this region occurred in 1611, 1896 and 1933. The 1896 Meiji-Sanriku earthquake was also a megathrust event which generated a destructive tsunami (Tanioka & Seno, 2001). The 1933 Sanriku-oki earthquake was also very destructive, and generated tsunami waves with heights ranging from 10 to 25 meters along the coast of Iwate prefecture of Honshu Island. Critical reviews and evaluations of historical events in Japan, as well as that of that of 11 March 2011 were undertaken by many researchers (Iida et al, 1967; Kanamori, 1971; Ammon et al., 2011; Koper et al 2011; Pararas-Carayannis 2011; Lay et al, 2011 a & b).

5.3a Review of Geotectonic Changes Associated with the 2011 Earthquake

Research on geotectonic changes indicates that the landmass of Honshu Island moved in an east-southeasterly direction, opposite to the direction of the under-thrusting forces. (Fig. 7).

Based on the Global Positioning System, the Geospatial Information Authority in Tsukuba, Japan, estimated that the Oshika Peninsula near the epicenter area moved by a little over 5 meters (17 feet) eastward and subsided by a little over 1 meter (4 feet). Additionally, the Geospatial Information Authority stated that there were land mass movements in many areas of Honshu, from the northeastern region of Tohoku to the Kanto region, including Tokyo. Slip and fault displacements were estimated to be up to 40 meters (Ammon et al., 2011).

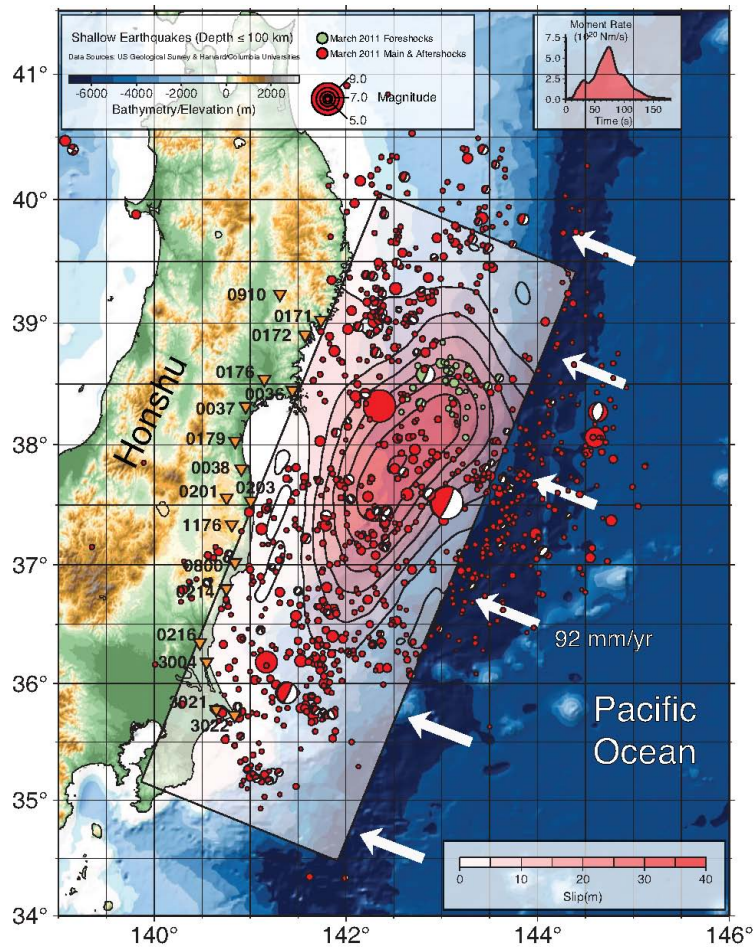


Fig. 7. Tsunami Generating Area showing the epicenter of the main earthquake on March 11, 2011 (After Ammon et al., 2011)

5.3b Review of the Mechanism of the 2011 Earthquake

In order to evaluate the tsunami source mechanism and the larger than expected heights of tsunami waves that were recorded or observed, an examination was undertaken of the above-stated seismo-tectonics of the region and of the earthquake's focal mechanism, energy release, rupture patterns and of the spatial and temporal sequencing and clustering of major aftershocks (Pararas-Carayannis 2013).

Based on this analysis, it was determined that the greater tsunami wave heights resulted from a combination of crustal deformations of the ocean floor due to up-thrust tectonic motions (Fig. 8). Additional uplift resulted from the quake's slow and long rupturing process, but also from large co-seismic lateral movements of surface seismic waves, which compressed and deformed the compacted sediments of the accretionary prism on the overriding plane.

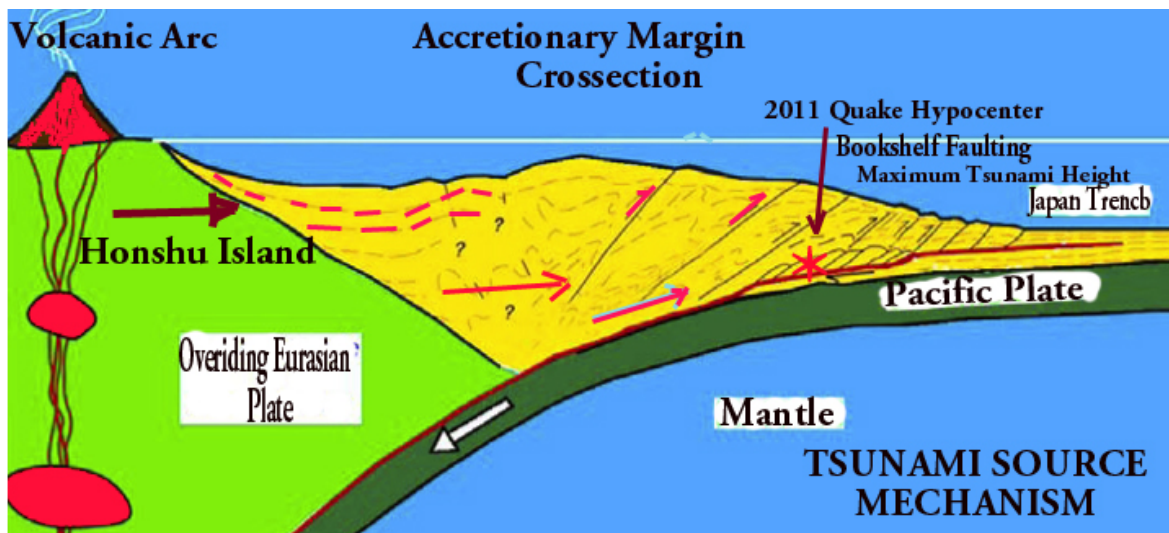


Fig. 8. Postulated Crosssection of the Accretionary Margin east of Honshu Island. Compression of the Sedimentary Prism and Subsequent Normal and Bookshelf Faulting contributed to the Tsunami's Source Mechanism and to greater Tsunamigenic Efficiency.

5.3c Review of the Mechanism of the 2011 Tsunami Generation

As mentioned in section 3.2 of this report - "Love" waves do not usually affect or enhance significantly tsunami wave heights, but the 2011 earthquake in Japan was an exception because the sedimentary prism east of Honshu Island was thick and highly fractured. Thus, the surface "Love" waves contributed to lateral compression of the sediments and, in combination with "Rayleigh" waves, contributed synergistically in both normal and bookshelf faulting.

At first, the sediments on the accretionary prism compressed elastically. However the elastic deformation was short-lived, as in the next few seconds the rupturing process nucleated existing normal faults on the continental shelf on both sides of the rupture. The reverse thrust motions and the lateral compression ruptured the sedimentary layers of the accretionary prism, which begun failing sequentially in a bookshelf fashion creating several parallel and en-echelon thrust faults (Pararas-Carayannis, 2011).

The deformation occurred randomly and non-uniformly along parallel normal faults and along the oblique, en-echelon faults to the earthquake's overall rupture direction - the latter failing in a sequential bookshelf manner with variable slip angles (Fig. 8). These additional deformational changes on the accretionary margin contributed to the generation of higher tsunami waves than those resulting only from the crustal displacements.

Vertical crustal displacements due to up-thrust faulting were estimated to be more than 10 meters. From lateral compression and folding of the sediments additional uplift was estimated to be about 7 meters - mainly along the leading segment of the accretionary prism of the overriding tectonic plate as shown in Figure 8 (Pararas-Carayannis 2013).

Tsunami generation was greater along the shallow eastern segment of the fault off the Miyagi Prefecture where most of the energy release of the earthquake and the deformations occurred, while the segment off the Ibaraki Prefecture - where the rupture process was rapid - released less seismic energy, compaction and deformation of sedimentary layers. Thus the tsunami was of lesser offshore height in the Ibaraki segment (Pararas-Carayannis 2013). As postulated in the following section, the spheroidal and the toroidal modes of the earth's oscillations did not affect in any significant way the height of the tsunami waves in the near-field area.

5.3d Evaluation of the Impact of Spheroidal and Toroidal Modes to the 2011 and 2004 Earthquakes' Crustal Displacement and Heights of Tsunami Waves

In searching the literature, no reference was found on whether the relative excitement of the earth's ultra-long period from the 2011 Japan earthquake was used to calculate more accurately – or to revise - its energy and moment magnitude. In the opinion of the author, because the spheroidal oscillations have very long periods, there was no contribution to the near-field tsunami heights. Also, the spheroidal oscillations could not have affected significantly the far-field tsunami wave heights.

In the following section there is a brief discussion of the excited "spheroidal normal modes" that followed the 26 December 2004 earthquake in Indonesia, which may have contributed somewhat to the puzzling higher tsunami-like wave (2.6 meter) recorded at Manzanillo, Mexico. Although some small contribution to tsunami generation by the earth's spheroidal normal modes is possible - and as previously stated - it is not expected to be significant because of the long periods of such oscillations.

6. IMPACT OF EARTH'S SPHEROIDAL OSCILLATIONS ON EARTHQUAKE DYNAMIC MOTIONS, ON TSUNAMI GENERATION, ON ATMOSPHERIC AND ON IONOSPHERIC FLUCTUATIONS

This section of the report reviews the postulated impact by the earth's excited spheroidal oscillations on dynamic motions of the lithosphere, on far-field tsunami generation, on interactions with the atmosphere and ionosphere and, finally, on whether there was an effect on the large magnetic field deviation of the earth, known as the "South Atlantic Anomaly".

6.1 Further Evaluation of the Impact of Spheroidal and Toroidal Modes on Earthquake Dynamic Motions and Tsunami Generation

Section 5.3d included a brief summary of postulated dynamic motions caused by spheroidal and toroidal modes from the 2011 and 2004 earthquakes. Section 5.3d dealt with the mechanism of tsunami generation and whether such excited oscillations contibuted to near and far field tsunami wave height enhancement. The following summary refers to the effects that sediments can have on earthquake rupture velocity and tsunami generation for other subduction zone regions - such as Makran in the North Arabian Sea,

the Sunda Trench segment in the Andaman Sea, and along the Mid-America Trench - which have been separately examined (Pararas-Carayannis, 1992; 2005; 2006).

For earthquake events in these regions, no correlation was made on whether the earth's long-period spheroidal oscillations were a partial causative factor for changes in the earthquake's rupture velocity, for the lateral or vertical displacement of surface or shallow subducted sedimentary layers, or for the augmentation in the height of tsunami waves. In all such regions of subduction, block motions of consolidated sediments were also associated with bookshelf-type of faulting, of surface sediments - which contributed to slow-rupturing, silent and deadly tsunami-earthquakes. Such was the case with the 2 September 1992 Nicaragua earthquake, with the exception that in this source region, only a limited amount of softer sedimentary layers existed in the accretionary prism. For this event, it was the slower rupture rate that contributed to the more significant tsunami that occurred.

Also, for the 1963 and 1975 earthquakes in the Kuril Islands, the large tsunami excitation was attributed to a slip in the accretionary sediment wedge (Fukao, 1979). Based on normal mode theory, Okal (1988), also showed that a tsunami source in the shallow sedimentary layer excites a much larger tsunami. The reason for such an outcome is that the extremely shallow block motions occur within shallow-subducted sediments where there is a lot of shear - thus the rupture is slower in speed. In all of such cases, the degree of sediment consolidation along a plate boundary appears to be a key factor in locking slippage on the megathrust region of the tectonic boundary, then releasing greater energy when the stress thresholds are exceeded. As already mentioned, such was the case of the 11 March 2011 earthquake in Japan, which was a megathrust event with the Pacific plate moving underneath the Eurasian plate (see Fig. 7).

As stated, great earthquakes generate strong shock waves and dynamic oscillations that affect the entire earth/water inter-phase in the oceanic regions where they strike. When they occur, the entire sea floor moves up and down from the shocks and rare-fractions that occur when massive amounts of energy are released deep in the earth. However, these are short period perturbations that occur rapidly over a time period that may last to a maximum of 60-80 seconds, a window of time may be too short for effective coupling with the water column in the source region. Thus, it is the net crustal displacements caused by an earthquake that contribute mainly to the major component of tsunami-genesis, and not the shorter time-period oscillations.

As for the effect of the earth's excited "spheroidal normal modes", such as those that followed the 26 December 2004 earthquake in Indonesia, it is possible that may have contributed somewhat to the higher tsunami-like observations observed and reported at distant stations. Although such additional contribution to tsunami generation by the earth's spheroidal normal modes is possible - and as previously stated - it is not expected to be significant because of the long periods of such oscillations.

As mentioned, what was a paradox for the 2004 tsunami was the unusually high tsunami-like wave of 2.6 meters recorded by the tide station at Manzanillo, Mexico, on the Eastern Pacific Ocean. Given the great distance of Manzanillo, Mexico, from the source region along the Sunda Trench in Western Sumatra, and the extensive chains of islands that

separate the Indian from Pacific Ocean, this wave height was unusual and raises questions on whether excitement of the longest period normal modes of the earth, such as the OS2 and OS3 may have been somewhat responsible for the far-field, higher tsunami recording in Manzanilo and elsewhere, although such contribution cannot be quantitatively measured and confirmed.

6.2 Evaluation of Coupling of the Earth's Spheroidal Oscillations with the Atmosphere and Ionosphere - Generation of Atmospheric and Ionospheric Oscillations

Regarding the coupling of the excited spheroidal oscillations with the atmosphere and the ionosphere, it is also difficult to determine quantitatively (Lognonné et al., 1998; Rhie & Romanowicz 2004). It is not known if any micro-barometers recorded this coupling of the Earth's cumulative (free and excited) spheroidal oscillations with the atmosphere following the 26 December 2004 great Sumatra earthquake, and whether distinct "normal modes" were detected. However, such combined coupling with the solid earth spheroidal motions would be expected to generate both atmospheric and ionospheric perturbations which could be measurable by microbarographs if they were available - and by means of the Doppler Effect (Artru et al., 2001, As mentioned in section 3 of this report, delayed ionospheric oscillations of Rayleigh waves were detected and recorded by high-frequency Doppler sounding techniques from the Kuril earthquake of 11 August 1969. In fact, the 10 MHz recording of Rayleigh waves was used to estimate the initial phase of the source of this earthquake (Furumoto, 1970), and as previously indicated, such timely estimation was considered to be a useful indicator of potential tsunami-genesis, and was recommended as early as 1970 for additional use by the Pacific Tsunami Warning Center (PTWC) in issuing tsunami warnings for the Pacific Ocean (Pararas-Carayannis, 2000 a, b, c).

6.3 Evaluation of Interactions of Spheroidal Oscillations with the Earth's Magnetic Field Anomalies and Exogenous Astronomical Influences.

In section 5.2 of this report about the great Earthquake of 26 December 2004, reference was made to the very large collection of split modes, not only of the spheroidal but also of the toroidal modes (Montagner & Roullet, 2004; Stein and Okal, 2005; <http://www.iris.iris.edu/sumatra/> ; Park EtAl, 2005). According to Park EtAl (2005), at periods greater than 1000 seconds, the Earth's seismic free oscillations have anomalously large amplitude when referenced to the Harvard Centroid Moment Tensor fault mechanism - which was estimated to be from 300 - to 500 - seconds surface waves. Such surface oscillations cannot affect the earth's magnetic field - which is determined by the movement of molten iron inside the earth's outer core, thousands of kilometers below the surface. The earth's internal density changes and movement of mass close to the earth's outer core and mantle interface, generates electrical currents which are mainly responsible for the earth's magnetic field and localized magnetic field anomalies. For example, when NASA satellites probing and measuring the Earth's gravity approach regions of higher density, they are pulled forward, at slightly higher speed and slow down somewhat when they reach regions of lesser crustal density.

Figure 9 is a NASA map of an area mainly affected by a growing lower magnetic anomaly, known as the “South Atlantic Anomaly”. This area stretches above the earth between South America and southwest Africa, as shown. The anomaly is not related to the earth’s normal or excited spheroidal oscillations. The most probable cause is the offset between the earth’s magnetic and rotational poles, the weakened magnetic field poles, and the energetic particles which penetrate closer to the earth’s surface in this region, where the Van Allen radiation belt is weaker. The Van Allen radiation belt is the protective zone which traps most of the energetically charged particles originating from solar winds, but not as effectively in the region of the aforementioned “South Atlantic Anomaly”. Apparently, the exogenous astronomical influence of solar winds are the cause of this South Atlantic Anomaly and the earth’s normal or excited spheroidal oscillations have nothing to do with it. The adverse effect of this magnetic anomaly in this particular region between South America and southwest Africa – and of concern for NASA - is only for their technological systems onboard satellites orbiting over this region.

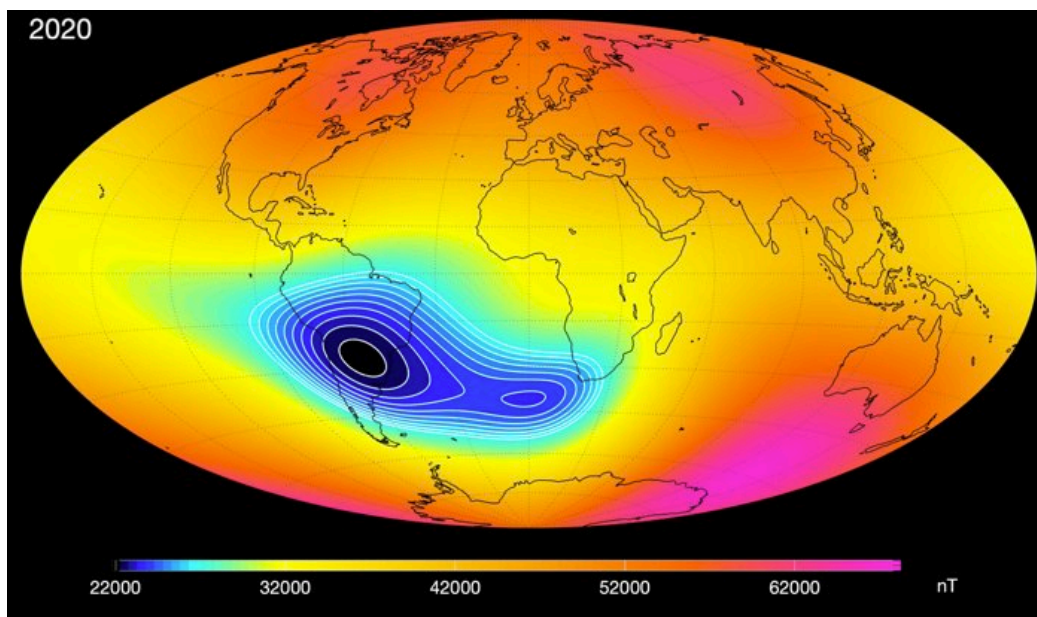


Fig. 9. Map of the region over South America and Southern Atlantic Oceans showing the growing lower magnetic anomaly, known as the “South Atlantic Anomaly”, where NASA satellites change their orbital velocities.

CONCLUSIONS

Based on the above review and preliminary evaluation, the following postulations, determinations and conclusions may be stated. Because of its rotation, the earth is aspherical and bulges in the equatorial zone region. The earth can be considered to be a mechanical system with a finite body of mass. All mechanical systems possess natural vibrations that can be excited by internal or external forces. The normal free oscillations of

the earth have been extensively studied by numerous researchers following recent earthquakes. Major and great earthquakes excite the earth's free oscillations with distinct frequencies which resonate over long periods of time. Coupling of these excited spheroidal oscillations result in distinct atmospheric as well as ionospheric perturbations of certain modalities and frequencies but, as expected, to a much lesser extent lithospheric oscillations. The coupling of excited spherical oscillations with the seas and oceans do not appear to enhance the height of tsunami waves in the near-field region of an earthquake, with a few exceptions, to far-field sea level fluctuations that resemble tsunami waves. However, atmospheric pressure disturbances from volcanic and meteorological events appear to couple more effectively with the atmosphere and the ionosphere to generate measurable perturbations, although their possible influence on free earth oscillations cannot be easily distinguished or quantitatively measured accurately.

No correlation was made on whether the earth's long-period spheroidal oscillations result in changes in the earthquake's rupture velocity, for the lateral or vertical displacement of surface or shallow subducted sedimentary layers, or for significant augmentation in the height of tsunami waves. However research on normal mode theory, indicated that a tsunami source in the shallow sedimentary layer excites a much larger tsunami. This occurs when extremely shallow block motions occur within shallow-subducted sediments where there is a lot of shear - thus where the rupture is slower in speed. In all of the cases that have been examined by researchers, the degree of sediment consolidation along a plate boundary appears to be a key factor in locking slippage on the megathrust region of the tectonic boundary, then releasing greater energy when the stress thresholds are exceeded. The "South Atlantic Anomaly", a magnetic field between South America and southwest Africa is not related to the earth's normal or excited spheroidal oscillations. Delayed ionospheric oscillations of Rayleigh waves from large earthquakes, have been detected and recorded by high-frequency Doppler sounding techniques.

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