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**OBJECTIVES:** The Tsunami Society publishes this journal to increase and disseminate knowledge about those aspects of natural hazards which involve the natural sciences. Natural hazards are those in which the opponent, gamewise, is not malevolent: natural sciences include, pragmatically, all but the social sciences.

Another objective is to promote the application of knowledge to mitigate the adverse effects of natural hazards.

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## GUEST EDITORIAL

The Tsunami Society has been created to satisfy the need for an organization focusing on those aspects of natural hazards which involve the natural sciences. (The natural hazards include hurricanes, earthquakes, tornadoes, tsunamis, etc.; the natural sciences include mathematics, physics, chemistry, etc.) This goal is complementary to that of studying the social aspects of natural hazards, which is already well organized. Since the phenomena are, by definition, natural in origin (here we use the ploy of defining the activities of man as being un-natural!), the importance of the natural sciences is obvious, and must be included in efforts to mitigate the hazard by modifying the source. This is complimentary to mitigating the effects by modifying the social responses to the natural event.

Tsunamis were, fortuitously, the phenomenon which prompted the formation of The Tsunami Society. Certainly other hazards are equally intriguing and challenging; but most importantly, there are similarities among natural hazards. There is the accumulation of energy, the storage of this energy--possibly with attenuation by diffusion, geometrical spreading, or by aging or absorption. The time dependence is of as much importance to the social aspect as to the natural sciences of the phenomenon. And engineering the modification of the hazards may require political and economic understanding and justification more than novel scientific research.

This journal is founded by The Tsunami Society to provide an outlet for articles on the natural-science aspects of natural hazards. Another benefit from this journal is that the publications on the natural science aspects of natural hazards now may be found in one place. The publications on many natural hazards, and especially on tsunamis, have been very diverse and often only in reports or unpublished manuscripts. Perhaps this reticence to publish in journals may be due to the recognition of the relatively low quality of the results of the research; but any such low quality is probably due to the difficulty of the problem and the scarcity of data on the phenomenon. Because of the rarity of the phenomenon, the observations or results are, in fact, probably more valuable than for some straightforward scientific work.

Because of the world-wide presence of inflation, the founding of a journal has required compromise, financially. This has prompted consideration and use of the modern methods of disseminating information. The increasing cost of materials and the escalating costs of distribution have made the trend to microforms apparent and inevitable. This is, perhaps, a characteristic to the evolution of a society. For example, the common hieroglyphics of the Egyptians decreased steadily in size over three thousand years, as may be easily noted from the tablets preserved in the Louvre, Paris. So this journal is a trend-setter, in using microforms. As this journal is prepared using a computer-based word processor, the evolution to

dissemination by electronic transmission should be straightforward.

The publication of a journal is the essence of any scientific society, but the nuances of the society are the meetings, at which the interactions between scientists bear fruit; albeit so often unplanned and serendipitously. For this reason, this Society will hold meetings biennially. These conferences will be in the years between the meetings of the Tsunami Conferences sponsored by the Tsunami Commission of the International Union of Geodesy and Geophysics (IUGG). For example, the IUGG holds meetings quadriennially--1979, 1983, etc.; the Tsunami Commission holds meetings biennially, in 1981, 1983, etc. The meetings of this Society (1982, 1984, etc.) provide the continuity between the biennial meetings. The meetings of this Society may be co-sponsored with other organizations: This is an invitation to other organizations to do so! The first conference scheduled for Honolulu, Hawaii, was in August, 1982. The title is "Physics and Mitigation of Natural Hazards." Those interested in presenting papers or attending these conferences should contact the Secretary of The Tsunami Society.

The diminution in the size of the writing used in a cultural society, over time, has been mentioned as a characteristic of the evolution of a society. Another characteristic of the evolution of a society is the entente of collective action. With the present stage of society, very little seems to be accomplished without such collec-

tive action. And such collective action is only justified when the objectives merit the effort of both organizing and working towards the objectives. Because of the importance of natural hazards to social history--but ask Caesar, Napoleon or Hitler about the weather over the English Channel, or book a ticket on the next Spanish Armada to England--the study of the natural-science aspects of natural hazards is extremely important to today's society. As it has been in the past, so it will continue to be in the future. This importance of natural hazards to mankind justifies, indeed, demands the creation and operation of organizations devoted to studying those natural hazards.

There is much competition for both your time and your money. To spend your time and money on the study of natural hazards can be a very discouraging activity because of the intrinsic difficulties of the phenomena; but this is offset by the satisfaction that comes from discovering a little more about the phenomena, especially when an improved predictive capability can be demonstrated. And the rarity of the occurrence of the phenomenon can be almost demoralizing; but this must be accepted as a challenging aspect.

Your participation and contributions to The Tsunami Society and this journal are invited and gratefully appreciated.

**PARAMETERS OF TSUNAMI WAVES  
IN THE SOURCE**

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**ABSTRACT**

Correlation relations of the displacement and tsunami wavelength in the source to the earthquake magnitude have been obtained that are useful for tsunami wave calculations.

## INTRODUCTION

The efficiency of hydrodynamic methods of calculation of possible damage caused by a tsunami wave on the shore depends in many respects on the knowledge of, "initial conditions", namely tsunami wave parameters in the source or in the open ocean at least. However, direct instrumental data on tsunami waves in the open ocean are not available. At the same time the theoretical methods of calculation of tsunami wave parameters in the source are not sufficiently reliable. As a rule, displacement and velocity fields at the bottom are taken as initial conditions (for tsunamis generated by earthquakes) but generally speaking they are unknown because instrumental measurements of real bottom displacements during an earthquake are not available (there is lack of data even on residual bottom deformation). Common solution of the hydrodynamic and elasticity equations is employed in an effort to relate the wave parameters in the source to measurable characteristics of a seismic process [A. S. Alekseev and V. K. Gus'akov, 1973; V. K. Gus'akov, 1974; V. K. Gus'akov, 1978] is undoubtedly, beneficial for elucidation of physics of generation of tsunami waves (particularly for definition of the type and orientation of the motion).

However, reliable comparison of the results of a theoretical model and the data on actual tsunami is not yet available and forecast formulae have not been deduced yet. Under these conditions it seems attractive to use the data on the former tsunamis for establishing the unknown

relation of tsunami wave parameters in the source to the basic characteristics of their cause, for example, the earthquake magnitude  $M$ , the wave field in the source being determined by the shore station data, i.e. by hydrodynamic methods. Lack of shore data and the difficulties of solving the inverse problem do not allow to establish exact relations between the characteristics under study. Nevertheless, there is a point in establishing such relations, if only on average, as it allows to put these data into a definite order, to reveal doubtful data and to make rough estimates necessary for practical forecasts.

Critical analysis of the available dependencies is given below and a number of new ones which allow to estimate tsunami wave parameters in the source by an earthquake magnitude have been suggested.

## I. WATER SURFACE DISPLACEMENT IN THE SOURCE

Estimation of the water surface displacement in the source reproducing the wave field from the shore data has been carried out by many authors using different methods [N. A. Shchetnikov, 1977; A. W. Garcia, J. R. Honston, 1976; K. Iida, 1963; T. Iwasaki, 1977; H. Watanabe, 1964]. The data obtained by different authors for a number of large tsunamis of the recent century is shown in Fig. 1. As is seen from the data by Iida, Hatori and Iwasaki the tsunami height in the source can amount to 10 meters. At the same time the estimations made by Watanabe

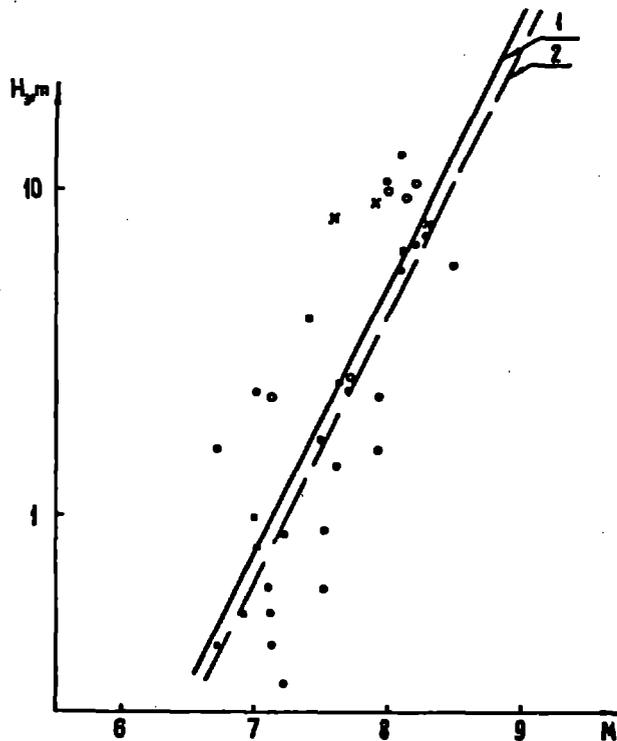


Fig. 1. The relation between the displacement in the tsunami source  $H_3$  and the earthquake magnitude  $M$

... - Hatori's data  
 ooo - Iida's data  
 xxx - Iwasaki's data

1 - the regression formula  
 $\log H_3 = (0,8 + 0,1) M - 5,6 + 1$

2 - Silgado's formula  
 $\log H_3 = 0,79 M - 5,7$

prove to be approximately ten times less. Meanwhile the latter have been widely used in schemes of tsunami-zonation.

Disagreement between the data by Watanabe and other authors appears to be due to the fact that Watanabe employed the ray method. The ray method is known to be used when the wave length is small as compared to the

typical dimensions of the bottom roughness. Clearly, this condition is not applicable on the shelf borders, which results in considerable reflection of wave energy (up to 80% in the Kurile-Kamchatskaya zone [S. L. Solov'ev, et al., 1977]). Hence for the inverse problem the "true" coefficient of the tsunami wave transformation must be lower than it follows from the ray method. We believe that this accounts for the small values of the water surface displacement in the source obtained by Watanabe. Larger values of wave lengths in the source are backed up by data on the motion of the sea bottom in the Sagami Bay (Japan) during the tsunami-genic earthquake 1.09.1923 obtained by the depth measurements made before and after the earthquake [V. V. Shuleikin, 1968]. On some sections the evaluation of the bottom amounts to 230 m, in others the bottom lowered by nearly 400 m. Analogous data is also available for land earthquakes during which the vertical displacements reach 10 m. [New catalogue of violent earthquakes on USSR from ancient times till 1975., 1977; I. D. Pon'avin, 1965]. This is indicative of possible wave heights of approximately 10 m in the source and hereafter we shall use the data obtained by Iida, Hatori and Iwasaki.

These authors' data are shown in Fig. 1. The regression formula derived on the basis of this information has the form:

\* The interval estimates of the regression equation coefficients given here and elsewhere below are in conformity with the confidence probability  $P = 80\%$ .

$$\log H = (0.8 \pm 0.1)M - 5.6 \pm 1.0 \quad (1)$$

where  $H_3$  is given in meters. This formula can be recommended for the analysis of tsunami originating in the vicinity of Japan and the USSR. It is interesting to mention the analogous formula by Silgado for tsunamis near the Western Coast of South America [E. Silgado, 1978].

$$\log H = 0.79M - 5.7 \quad (2)$$

As is seen from Fig. 1 the two formulae give similar values of the tsunami wave height for  $6.7 < M < 8.5$ .

## II. TSUNAMI WAVELENGTH IN THE SOURCE

The dimensions of the tsunami source are usually determined by two methods; by solving the reverse problem for the wave equation (in fact only by means of drawing rays and estimating the travel time from origin to different sections of the shore) or by assessing the source dimensions of the earthquake (which corresponds to the area of the aftershock activity). Numerous calculations have shown that the tsunami source has approximately the same dimensions as the source of the earthquake [S. L. Solov'ev, 1968].

In the general case the source can be approximated by an ellipse with semiaxis  $a$  and  $b$ . The tsunami behaviour at large distances from the source depends essentially on its area or mean radius  $R = \text{SQRT}(ab)$ . Often only the tsunami source area  $S$  is given. The data on tsunami source area obtained by dif-

ferent authors is given in Table 1. The values of the mean source radius calculated from  $R = \text{SQRT}(S / \pi)$  or by the length of semi-axes of the ellipses are also given here. These data are shown in Fig. 2. The empirical

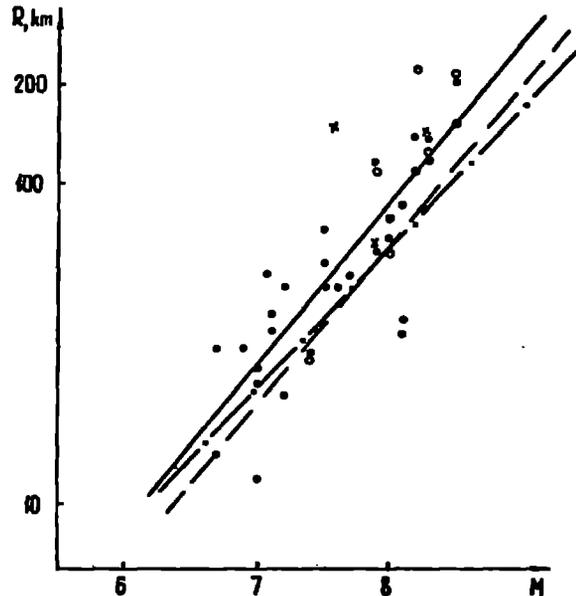


Fig. 2 The relation between the tsunami source radius  $R$  and the earthquake magnitude  $M$

- ... - Hatori's data
- ooo - Solov'ev's data
- xxx - Iwasaki's data
- OOO - Watanabe's data

\_\_\_\_\_ the regression formula:

$$\log R = (0,50 + .7)M - 2,1 + .6$$

Iida's formulae:

- $\log R = 0,5 M - 2,2$
- .....  $\log R = 0,45 M - 1,8$

relations obtained by Iida in 1958 and 1963 are also represented here:

$$\begin{aligned} \log R &= 0.5 M - 2.2 \\ \log R &= 0.45M - 1.8 \end{aligned} \quad (3)$$

TABLE 1

## DATA ON TSUNAMI SOURCE PARAMETERS

No.	Date	Location	Magnitude	Level Displacement in the Source in Meters					Dimensions of the Source						
				Iida	Watanabe	Hatori	Iwasaki	Watanabe	Hatori	Iwasaki	Solov'ev				
1.	15.06.1896	Sanriku	7.8	-	-	-	8.15	-	-	-	-	15/1.8	1.44	-	-
2.	1.09.1923	Kanto	7.9	-	-	1.6	-	-	-	12	0.62	-	-	-	-
3.	27.05.1928	Iwate	7.0	-	-	0.8	-	-	-	2.3	0.27	-	-	-	-
4.	9.03.1931	E.Aomori	7.0	-	-	1.0	-	-	-	0.4	0.12	-	-	-	-
5.	3.03.1933	Sanriku	8.3	6.5	0.95	8.0	7.54	48	1.23	57	1.35	2.1/0.9	1.41	1.5/1	1.2
6.	19.09.1933	Miyagi	7.1	-	-	0.6	-	-	-	3.8	0.35	-	-	-	-
7.	13.10.1935	Iwate	7.2	-	-	0.3	-	-	-	7.4	0.49	-	-	-	-
8.	2.11.1936	Sanriku	7.7	7.1	-	-	-	-	-	-	-	-	-	-	-
9.	23.05.1938	Ibaraki	7.1	2.3	-	0.5	-	-	-	4.8	0.39	-	-	-	-
10.	5.11.1938	Ibaraki	7.7	-	-	2.4	-	-	-	8.8	0.53	-	-	-	-
11.	5.11.1938	Ibaraki	7.6	-	-	1.4	-	-	-	7.4	0.49	-	-	-	-
12.	6.11.1938	Fukushima	7.5	2.6	-	0.9	-	-	-	7.0	0.47	-	-	-	-
13.	7.11.1938	Fukushima	7.1	-	-	0.4	-	-	-	8.5	0.52	-	-	-	-
14.	14.11.1938	Fukushima	7.0	-	-	2.4	-	-	-	1.6	0.23	-	-	-	-
15.	22.11.1938	Fukushima	6.7	-	-	1.6	-	-	-	3.1	0.31	-	-	-	-
16.	1.05.1939	Oga	6.7	-	-	0.6	-	-	-	0.6	0.14	-	-	-	-
17.	2.08.1940	W.Hokkaido	7.5	-	-	0.6	-	-	-	1.6	0.71	-	-	-	-
18.	7.12.1944	Tonankai	8.0	10	-	1.1	-	-	-	14	0.67	-	-	1/0.3	0.6
19.	21.12.1946	Hankaido	8.1	9.5	-	5.9	-	-	-	4.6	0.38	-	-	-	-
20.	4.03.1952	Tokachi-Oki	8.1	6.5	0.54	12.8	-	23	0.85	37	0.34	-	-	-	-
21.	4.11.1952	Kamchatka	8.2	10.7	0.78	7.0	-	38	1.1	60	1.38	-	-	4/1.3	2.23
22.	25.11.1953	Boso-Oki	7.5	-	0.06	3.3	-	-	-	10	0.56	-	-	-	-
23.	6.11.1958	Iturup	8	-	-	-	-	19	0.78	-	-	-	-	1/-	-
24.	22.05.1960	Chili	8.5	-	-	8	-	71	1.5	138	2.09	-	-	5/1	2.12
25.	27.02.1961	Fiuganada	7.2	-	-	0.9	-	-	-	1.6	0.22	-	-	-	-
26.	13.10.1963	Urup	7.9	-	-	2.3	-	-	-	42	1.15	-	-	210.5	1.0
27.	7.05.1964	W.Aomori	6.9	-	-	0.5	-	-	-	3	0.31	-	-	-	-
28.	16.06.1964	Niigata	7.4	-	-	3.9	-	-	-	2.7	0.29	-	-	0.4/0.2	0.28
29.	16.05.1968	Tokachi-Oki	7.9	-	-	-	9.18	-	-	-	-	0.8/0.5	0.63	-	-

where R is given in kilometers. The regression dependencies obtained from the data listed in Table 1 is of the form:

$$\log R = (0.50 \pm 0.07)M - (2.1 \pm 0.06) \quad (4)$$

As seen from Fig. 2 these formulae give approximately the same values of the source radius for  $6.7 < M < 8.5$ .

Generally speaking the source radius does not correspond to the wavelength leaving the tsunami (the proportionality of these values may be expected for example when supposing a piston motion of the bottom). In [N. R. Mirchina, E. N. Pelinovsky, 1980] the wavelength source radius relation has been shown:

$$\lambda_3 = 2.8R \quad (5)$$

Using Equ. (5), we finally obtain the expression for the tsunami wavelength in the source:

$$\log \lambda_3 = 0.5M - 1.7 \quad (6)$$

The values of  $\lambda_3$  and  $H_3$  used in the schemes of tsunami zonation are given in Table 2. As

is seen from Figure 3 these schemes are based on source parameters taking no account of their dependencies on the earthquake magnitude. At the same

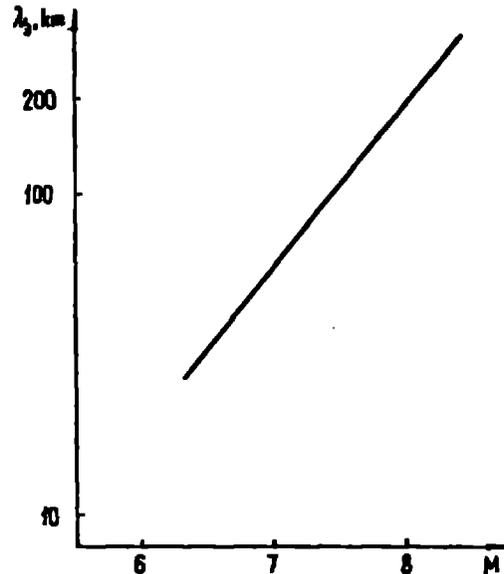


Fig. 3. The relation between the the tsunami wavelength in the source and the earthquake magnitude.

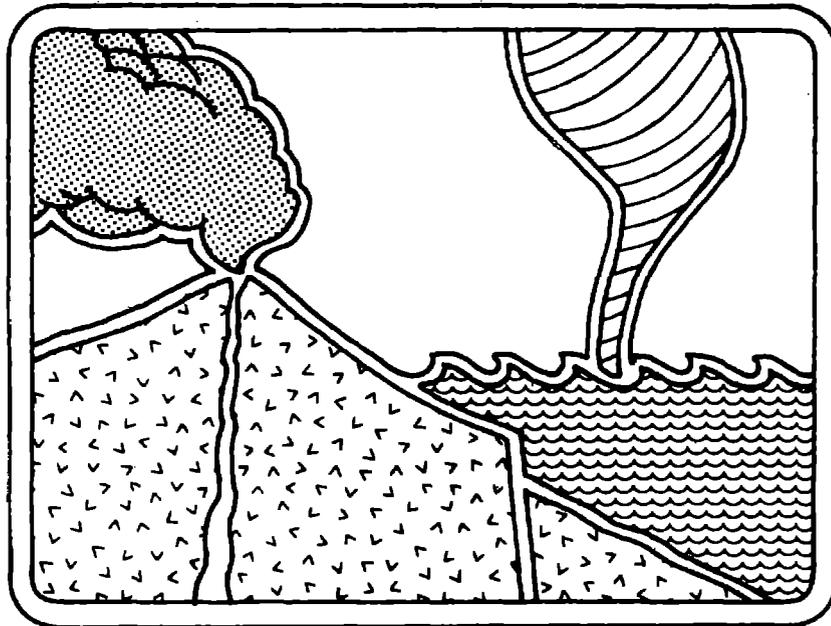
time Equations (1) and (6) enable us to estimate tsunami wave parameters in the source over a wide range of earthquake magnitudes.

TABLE 2

Reference	Solov'ev S.L. et al.	1978 Atlas	Brandsma, M. et al. Garcia, A.W. & Houston, J.R.
Level displacement in the source	2m	1m M < 8 2m 8 <= M <= 8.5 4m 8.5 < M	9.15 (30 feet)
Wavelength in the source		Source form	
		Line 90km      120km	Ellipse with dimensions 1066km    480km

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## **A MODEL OF THE 1975 HAWAII TSUNAMI**

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### **ABSTRACT**

The Hawaii tsunami of November 29, 1975 was calculated using a linear shallow-water-wave code assuming various source models. It was also modeled using a three-dimensional code SOLA-3D for solving the incompressible, nonlinear Navier-Stokes equations. The observed tsunami wave profile near the source was a second wave larger than the first wave. This could be assumed as a step function for the source motion. The observed wave profile may be approximated by the nonlinear calculation without a source motion step function.

## INTRODUCTION

The tsunami of November 29, 1975 has been investigated by Loomis. He described the observed runup heights in [H. G. Loomis, 1975] and a numerical study of the tsunami source in [H. G. Loomis, 1978].

The tsunami was generated by an earthquake near the Hawaii Volcanoes National Park with a magnitude of 7.2 on the Richter scale. Near the source, the first wave was smaller than the second. Coincident with the earthquake was considerable subsidence (up to 3 meters) of the shoreline.

Loomis, in [H. G. Loomis, 1978], examined a model of the Southeastern Coast of Hawaii. The bottom slopes seaward at a ratio of 1:15 until it reaches a constant depth of 6000 meters. The sources examined by Loomis included both initial uplifts and depressions and he reported that such source motions would not generate the essential features of the tsunami; that is, a second wave larger than the first.

We have used the SWAN code described in [C. L. Mader, 1976 and C. L. Mader, 1974] to solve the long wave, shallow water equations and examine the tsunami generation problem. We confirmed Loomis' calculated results using our linear shallow-water-wave code. We have used the SOLA-3D code that solves the nonlinear, three-dimensional, incompressible Navier-Stokes equations to model the tsunami. Close to the source of the wave the second wave was calculated to be larger than the first wave with a

source motion of an initial depression of the ocean surface.

## I. THE CALCULATED LINEAR SHALLOW-WATER-WAVE LENGTHS

Our model is essentially identical to that described by Loomis in [H. G. Loomis, 1978]. A 40-by-69 rectangular region of 207 km along the coast and 120 km seaward is described using a mesh of 3 km by 3 km. The bottom slopes at a ratio of 1:15 until it reaches a depth of 6 km. The source is a bottom slope subsidence programmed to vary with time. The source is 30 km wide, of which half is included in the calculation and is separated from the other half by a reflective boundary as shown in Fig.1.

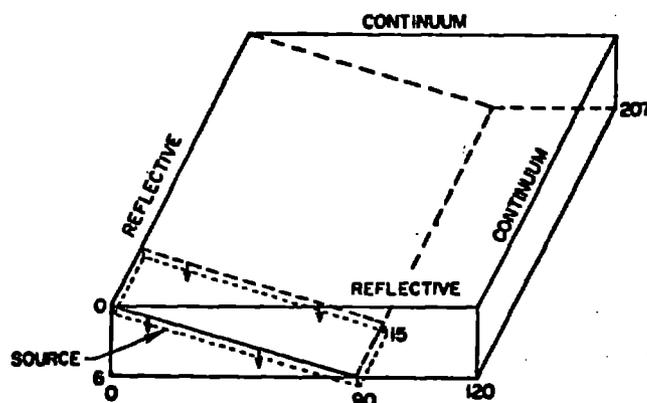


FIG. 1. Sketch of model used to numerically simulate the tsunami generation.

The calculated wave profile is shown at various locations along the shoreline as a function of time in Fig. 2 for a source displacement of 3 meters and in Fig. 3 for a source displacement of 1 meter followed by an addi-

tional 2-meter displacement 10 minutes later. Surface profiles are shown in Fig. 4.

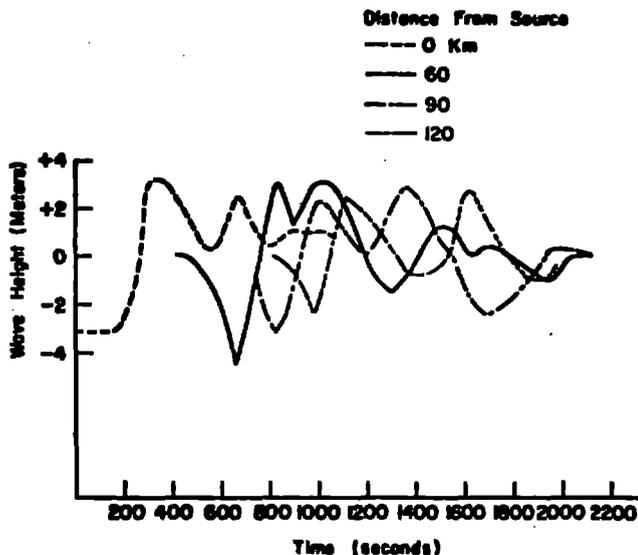


FIG. 2. Shoreline waveheights for linear shallow-water-wave model resulting from an initial source displacement of 3 meters.

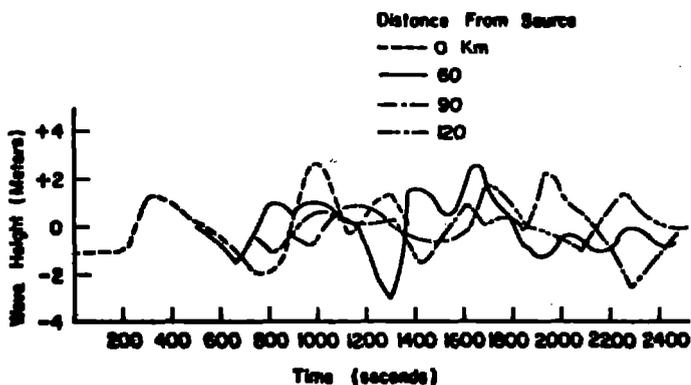


FIG. 3. Shoreline waveheights for linear shallow-water-wave model resulting from an initial source displacement of 1 meter followed by an additional 2-meter displacement 10 minutes later.

The observed larger second wave can be reproduced by a source that has its displacement change with time. Such a possibility was suggested by [M. Ando, 1978] who suggested that the earthquake was a rather slow rupture lasting 100 seconds; however, the displacement change required by the model is of longer duration and two fast ruptures.

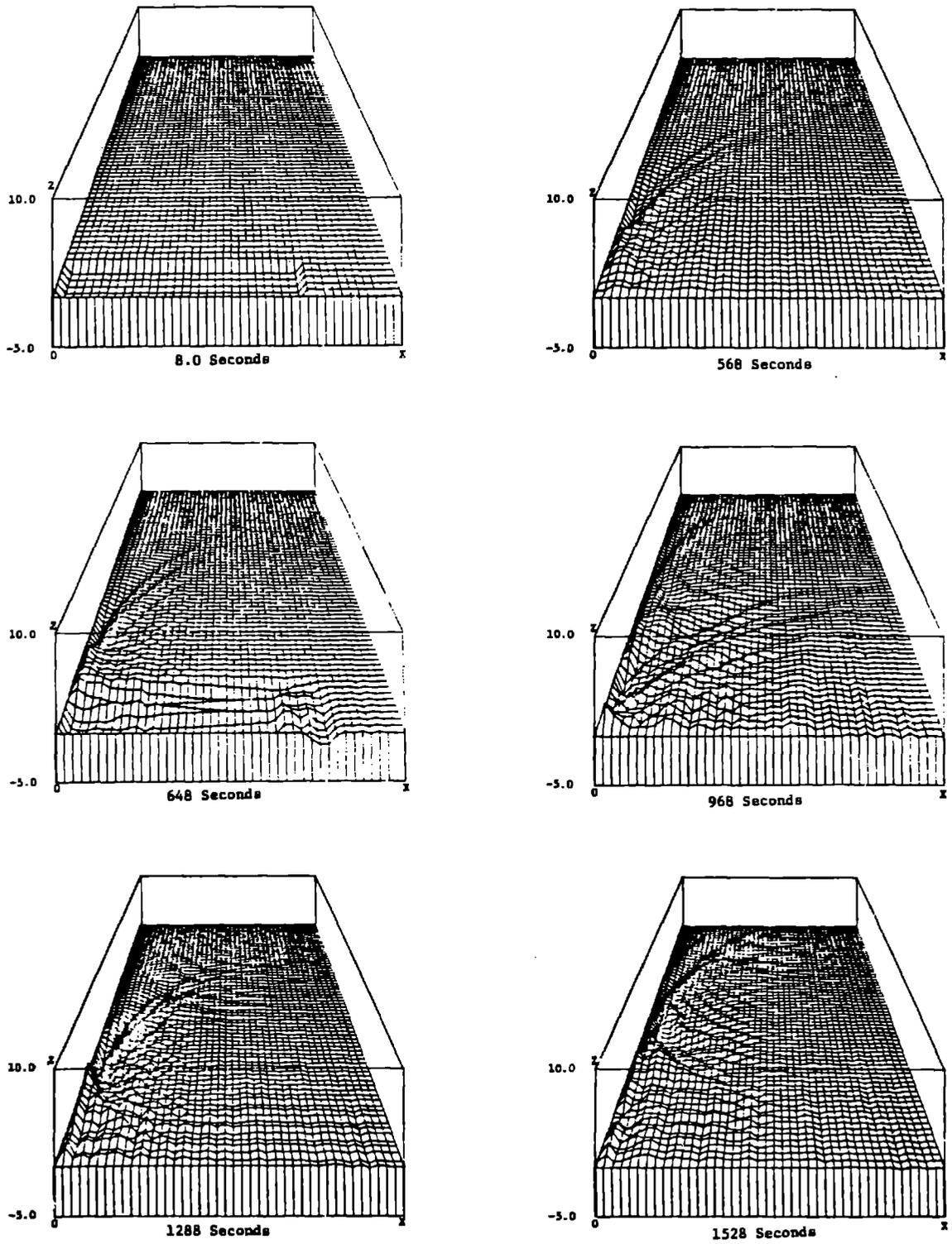
## II. THE CALCULATED NONLINEAR WAVE RESULTS

Three-dimensional, time dependent, nonlinear, incompressible viscous flow was calculated for the model shown in Fig. 1 using the SOLA-3D code.

The SOLA-3D code is a three-dimensional version of the two-dimensional SOLA code described in [C. W. Hirt and B. D. Nichols and N. C. Romero, 1975]. The program has evolved from the marker-and-cell (MAC) finite difference technique which uses pressure and velocity as primary dependent variables. A variable mesh capability has been included to improve the numerical resolution. The surface height of the center of each cell is computed each cycle according to the kinematic equation

$$\frac{\partial H}{\partial t} + U \frac{\partial H}{\partial x} + V \frac{\partial H}{\partial y} = W,$$

similar to that described in [C. W. Hirt and B. D. Nichols and N. C. Romero, 1975] for the SOLA-SURF version of the SOLA code. The Navier-Stokes equations for incompressible viscous fluid flow are



**Fig. 4.**  
Surface profiles for linear shallow-water-wave model as a function of time, resulting from an initial shore displacement of 1 meter followed by an additional 2-meter displacement 10 minutes later.

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = 0$$

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} + W \frac{\partial U}{\partial z} = - \frac{\partial P}{\partial x} + g_x + \nu \left( \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} \right)$$

$$\frac{\partial V}{\partial t} + U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} + W \frac{\partial V}{\partial z} = - \frac{\partial P}{\partial y} + g_y + \nu \left( \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \right)$$

$$\frac{\partial W}{\partial t} + U \frac{\partial W}{\partial x} + V \frac{\partial W}{\partial y} + W \frac{\partial W}{\partial z} = - \frac{\partial P}{\partial z} + g_z + \nu \left( \frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2} + \frac{\partial^2 W}{\partial z^2} \right)$$

where

U, V, W are velocity components in the x, y, z directions,  
 t is time,  
 P is pressure,  
 $g_x, g_y, g_z$  are x, y, z components of gravity, and  
 $\nu$  is the kinematic viscosity coefficient.

The equations are solved using the finite difference technique described in [C. W. Hirt and B. D. Nichols and N. C. Romero, 1975].

The geometry of the model used to calculate the tsunami is shown in Fig. 1. The mesh used in the calculation had 20 cells in the x-direction, 25 cells in the y-direction, and 18 cells in the z-direction. The 20 cells in the x-direction were 6 km wide. The 18 cells in the z-direction starting at the ocean floor were 100 meters high for the first two cells, and 400 meters thereafter. The water depth was 6000 meters and the surface was located at the center of cell 17. The 25 cells

in the y-direction starting at the source were 3.0 km for the first 5 cells which described the source (15 km wide). The remaining cell widths were 5.75, 6.16, 6.56, 6.97, 7.37, 7.78, 8.18, 8.59, 9.0, 9.4, 9.8, 10.2, 10.6, 11.0, 11.4, 11.8, 12.2, 12.6, 13.0, and 13.5 km, for a total of 206.8 km.

The viscosity coefficient was  $2.0 \text{ g-sec}^{-1} \text{ -m}^{-1}$  (0.02 poise). The gravity constant,  $g_z$ , was  $-9.8 \text{ m-sec}^{-2}$ , and  $g_x$  and  $g_y$  were 0.0. The time step for the calculation was 5 seconds. The tsunami source was modeled by a 3-meter-deep depression or elevation of 90 by 15 km of the water surface, as shown in Fig. 1.

The calculated wave profiles are shown at various locations along the shoreline as a function of time in Figs. 5 for a source of 3-meter depression of the water surface. The calculation for a 3-meter uplift gave mirror images of the profiles for the 3-meter depression of the water surface. Surface profiles are shown in Fig. 6.

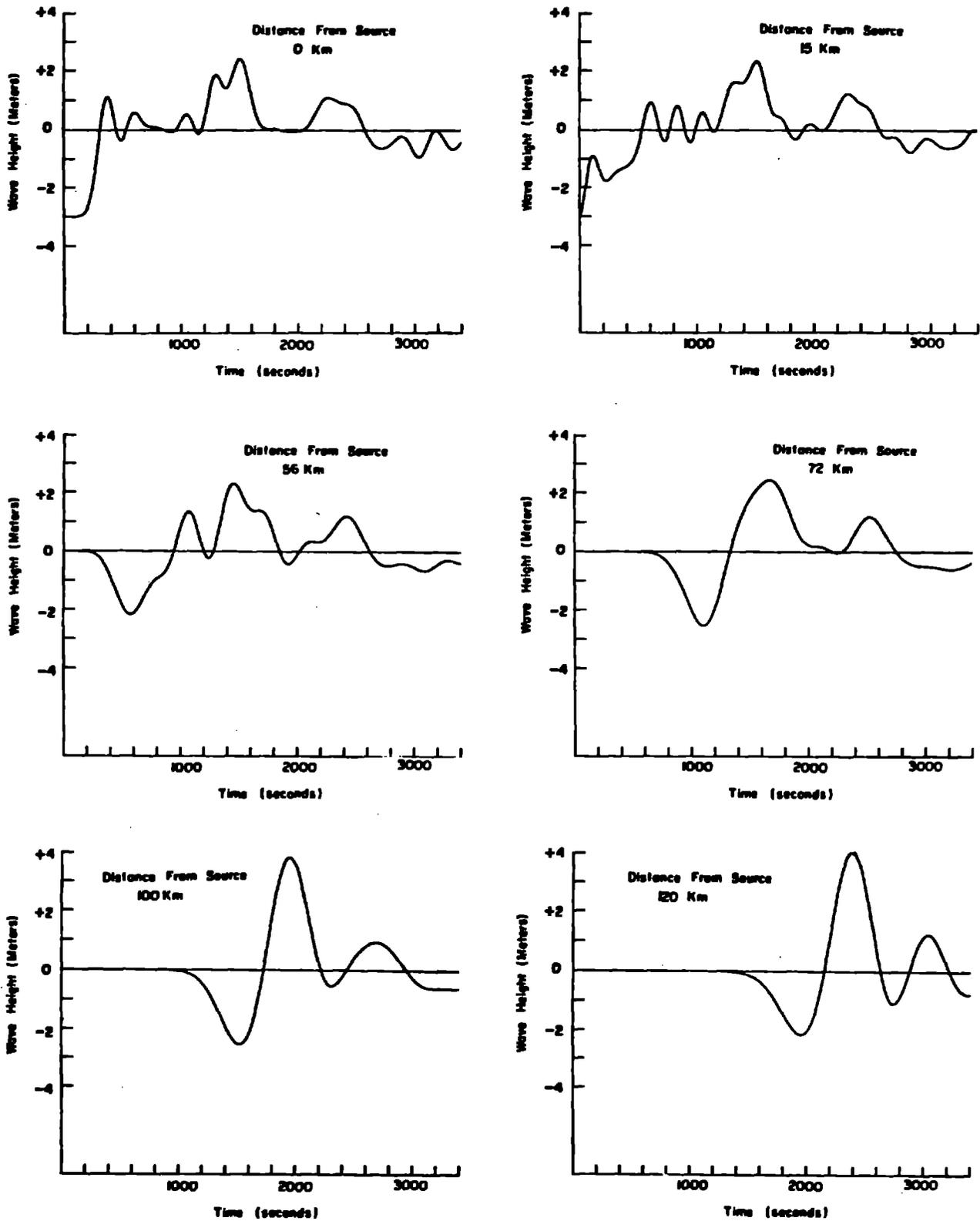
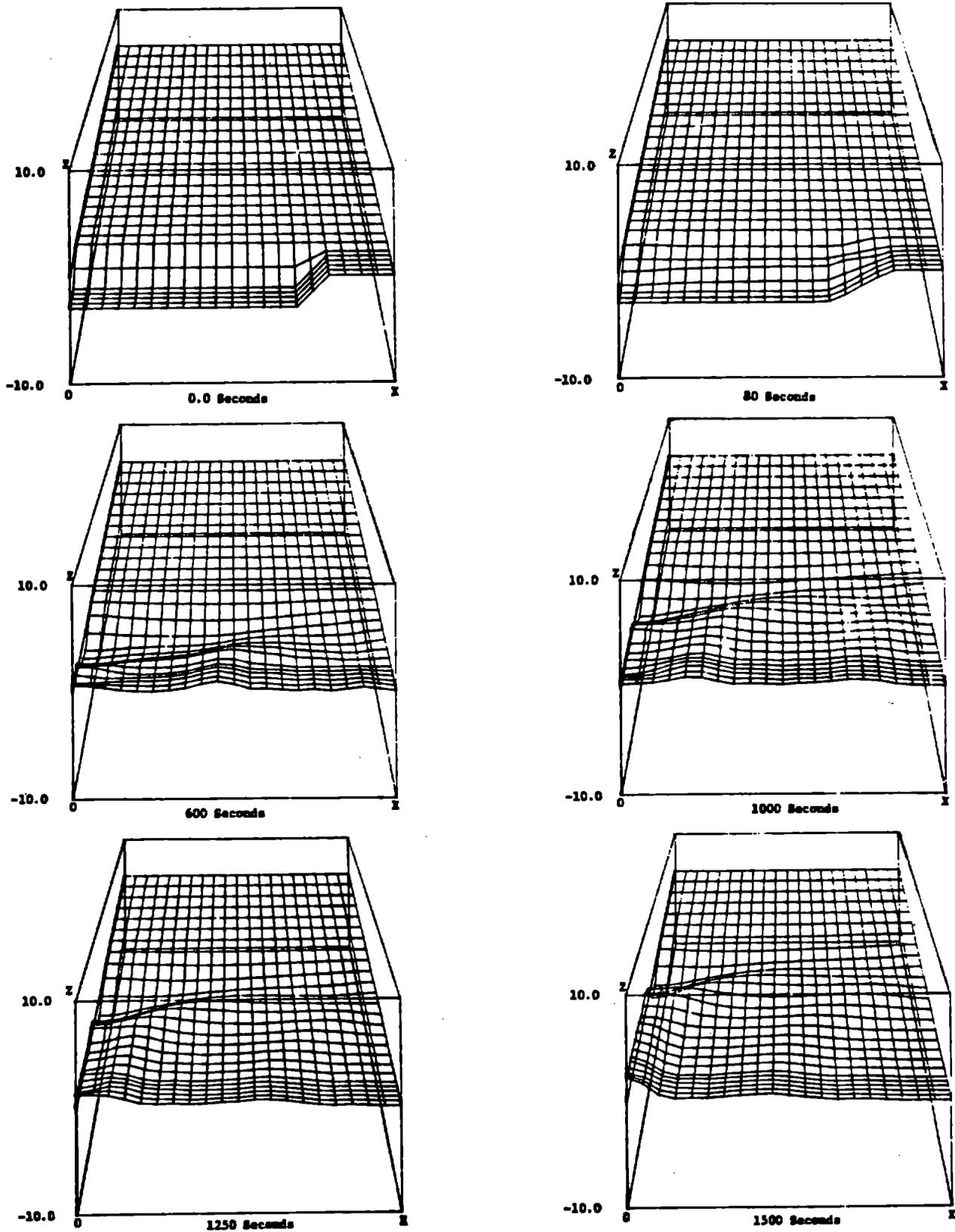


Fig. 5. Shoreline wave heights for the nonlinear model as a function of time, resulting from an initial source of a 3-meter depression of the water surface.



**Fig. 6.**  
 Surface profiles for nonlinear model as a function of time resulting from an initial source of a 3-meter depression of the water surface.

### III. CONCLUSIONS

The observed tsunami wave profile of the 1975 Hawaii tsunami near the source of the second wave larger than the first wave may be reproduced by a nonlinear incompressible calculation. The observed profile could not be reproduced by a linear shallow-water-wave calculation without introducing a step function for the source motion.

The nonlinear waves were slower than the linear waves and the interaction was much more complicated. Close to the source of motion the first wave was generated by the cavity being filled along the shoreline. The second wave was generated by the cavity being filled from the deep water end. The two waves interacted along the shoreline resulting in the second wave being the largest wave with a velocity greater than the first wave. The second wave overtook the first wave at later times and greater distances from the source.

### ACKNOWLEDGEMENTS

We recognize the contributions of Dr. H. Loomis of JIMAR, who suggested we study the 1975 Tsunami source problem, and the critical review by Dr. E. Bernard of the Pacific Marine Environmental Laboratory and Dr. W. Adams of the Hawaii Institute of Geophysics. We also recognize helpful discussions with C. W. Hirt of Flow Science, Inc., and James D. Kershner and Allen L. Bowman of the Los Alamos National Laboratory.

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#### Note:

- One reviewer suggests that readers also see:
- Skalarz, M. A. and L. Q. Spielvogel and H. G. Loomis, 1979. "Numerical Simulation of the 29 Nov. 1975 Island of Hawaii Tsunami by the Finite Element Method," Journal of Physical Oceanography, vol. 9, no. 5, pp. 1022-1031

**NONLINEAR AND DISPERSIVE EFFECTS  
FOR TSUNAMI WAVES  
IN THE OPEN OCEAN**

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**ABSTRACT**

The influence of nonlinear and dispersive effects for tsunamis, that occurred near the Pacific shore of the USSR and Japan, are performed. It is shown, that these effects can be essential for tsunami waves crossing the Pacific Ocean.

**INTRODUCTION**

The influence of nonlinearity and dispersion on tsunami wave propagation was repeatedly considered in literature. Practically all numerical calculations of tsunami waves performed within the limits of linear and nonlinear shallow water equations show that nonlinearity can be neglected, although the calculations are performed for restricted water areas [A. S. Alexeev et al., 1978; V. G. Bukhteev and N. L. Plink, 1978]. Analytical estimates given in [J. L. Hammack, 1973; H. Nagashima, 1977; E. N. Pelinovsky, 1977] for "conditional" tsunami wave parameters are rough enough and do not enable to make univalued conclusions. That is why, for solving this problem completely, we shall use factual material on past tsunamis to make statistically significant conclusions.

**I. FORMULATION OF THE PROBLEM**

Since the amplitude is small, the theoretical analysis of nonlinear and dispersive effects for tsunami waves is carried out in the framework of the Korteweg-de Vries equation, that has already been used in connection with the tsunami problem [J. L. Hammack, 1973; H. Nagashima, 1977; L. A. Ostrovsky and E. N. Pelinovsky, 1977; E. N. Pelinovsky, 1977]

$$\frac{\partial \eta}{\partial t} + \sqrt{gh} \left(1 - \frac{3\eta}{2h}\right) \frac{\partial \eta}{\partial x} + \frac{h^2 \sqrt{gh}}{6} \frac{\partial^3 \eta}{\partial x^3} = 0 \quad (1)$$

where  $\eta$  is the water level dis-

placement,  $h$  is the ocean depth,  $g$  is the gravity acceleration.

Stationary single waves (solitons) play an essential role in solving the Korteweg-de Vries equation. The form of such a wave is described by the expressions:

$$\eta = H \operatorname{sch}^2 \sqrt{\frac{3H}{4h}} \frac{x - \sqrt{g(h+H)}t}{h} \quad (2)$$

and determined only by the height  $H$  (the other parameter is the crest location at the initial moment.). It is essential that only the elevation wave is a stationary one. The soliton length determined by the level 0.5 is equal to

$$\lambda = 2h \sqrt{\frac{4h}{3H}} \ln(1 + \sqrt{2}) \approx 2h \sqrt{h/H} \quad (3)$$

The values of  $\lambda$  for typical  $h$  and  $H$  in the open ocean are listed in Table 1.

Table 1  
Soliton Length in km

$\frac{h}{\text{km}} \backslash \frac{H}{\text{m}}$	0.5	1	2	3	4	5	10
0.5	31	22	16	13	11	10	7
1	89	63	45	36	31	28	20
2	253	179	126	103	89	30	56
3	465	330	230	190	164	147	104
4	715	506	357	292	253	226	160
5	1000	707	505	408	353	316	223

The well-known results of the general theory of the Korteweg-de Vries equation are briefly the following: an arbitrary-shape impulse in the general case, breaks into solitons and quasi-periodic waves [V. I. Karpman, 1973; G. Whitman, 1974]. The characteristics of the process are determined by the so called similarity parameter that coincides with the Ursell parameter:

$$U = \frac{H_s \lambda_s^2}{2 h_s^3} \quad (4)$$

( $H_s$  is the water level displacement in the source,  $\lambda_s$  is the wavelength in the source,  $h_s$  is the ocean depth in the source), which as it is easily seen from Equ. 1 is the ratio of the nonlinear term to the dispersion one (for the soliton  $U = 4$ ). The Ursell parameter is actually proportional to the square of the ratio of the initial disturbance length to the soliton length with the same amplitude.

## II. ESTIMATION OF THE URSELL PARAMETER

Let us estimate the Ursell parameter for actual tsunamis. The data on these tsunamis (the epicenter coordinates, the magnitude  $M$ , the ocean depth in the source  $h_s$ , the water level displacement in the source  $H_s$  and the source radius  $R$ ) taken from [T. Hatori, 1966; T. Hatori, 1973; T. Iwasaki, 1977; H. Watanabe, 1964] are given in Table 2. We calculated the

tsunami wavelength in the source by the formula  $\lambda = 2.8R$  ( $R$  is the mean source radius determined by the source area given in [T. Hatori, 1966; T. Hatori, 1970]) obtained in [N. R. Mirchina and E. N. Pelinovsky, 1981 (to be published)] on a basis of statistical treatment of data on actual tsunamis.

The results of the calculation of the value  $Uh_s^3$  for a number of past tsunamis are presented in Table 2. The correlation analysis of these data has shown the presence of linear regression relation of the value  $\log Uh_s^3$  and the earthquake magnitude  $M$ :

$$\lg Uh_s^3 = (1.6 \pm 0.2)M - 11 \pm 2 \quad (5)$$

Here  $h_s$  is expressed in kilometers. The interval estimates of the regression coefficients are in conformity with the confidence probability  $P = 80\%$ . The correlation field and regression relation Equ. (5) are given in Fig. 1, which allows to determine the Ursell parameter by the earthquake magnitude and the ocean depth in the source. The Ursell Parameter values calculated for a number of past tsunamis are also given in Table 2 and Fig. 2. The formula for the Ursell parameter follows from Equ. (5) (Fig. 2):

$$\lg U = -3 \lg h_s + 1.6 M - 11 \quad (6)$$

TABLE 2

Estimates of Nonlinear and Dispersive Effects in Tsunamis Caused by Underwater Earthquakes

No.	Earthquake					$h_b$ km	$H_b$ m	R km	$\lambda_b$ km	$U_{H_b}^2 = \frac{H_b \lambda_b^2}{2}$ $km^3$	U	max $L_H$ thousand km	$L_D$ thousand km	Non-Dispersion	
	Date (GMT)	Epicenter		Location	M									Non- linearity + important - unimportant	Dispersion + important - unimportant
		Latitude N°	Longitude E°												
1	15.06.1896	39.6	144.2	Sanriku	7.6	5	8.3	144	403	665	5.3	165	-	-	
2	27.05.1928	40.0	143.2	Iwate	7.0	1.2	0.8	27	76	2.3	1.3	-	18	-	+
3	9.03.1931	41.2	142.5	E.Aomori	7.0	0.3	1.0	12	34	0.6	21	6.7	-	+	-
4	2.03.1933	39.1	144.7	Sanriku	8.3	5.5	7.8	136	378	542	3.3	195	-	-	-
5	19.06.1933	38.1	142.4	Miyagi	7.1	1.5	0.8	38	98	2.9	0.9	-	25	-	-
6	13.10.1935	40.0	143.6	Iwate	7.2	1.8	0.3	49	137	2.8	0.5	-	48	-	-
7	2.11.1936	38.2	142.2	Miyagi	7.7	0.7	7.1	30	84	25	73	6	-	+	-
8	23.05.1938	36.7	141.4	Ibaraki	7.1	0.5	2.3	39	109	13.7	109	16	-	+	-
9	5.11.1938	37.1	141.7	Ibaraki	7.7	1	2.4	53	148	26.3	26.3	42	-	-	-
10	5.11.1938	37.2	141.7	Ibaraki	7.6	1	1.4	42	118	9.7	9.7	56	-	-	-
11	5.11.1938	37.5	141.8	Fukushima	7.5	1	2.0	47	132	22.3	22.3	34	-	-	-
12	6.11.1938	37.0	141.7	Fukushima	7.1	1	0.4	62	145	4.2	4.2	243	-	-	-
13	13.11.1938	37.0	141.5	Fukushima	7.0	0.6	2.4	23	65	5.1	23.4	11	-	+	-
14	22.11.1938	37.0	141.8	Fukushima	6.7	1	1.6	31	87	6.1	6.1	36	-	-	-
15	1.05.1939	40.0	139.8	Oga	6.7	0.2	0.6	14	39	0.5	57	8.7	-	+	-
16	1.08.1940	44.1	139.5	W.Hokkaido	7.5	1	0.8	71	200	12	12	223	-	-	-
17	7.12.1944	33.7	136.2	Tonankai	8.0	1.8	10.0	87	188	177	30	23	-	-	-
18	20.12.1946	33.0	135.6	Nankaido	8.1	2	9.8	38	106	53	6.7	14.7	-	+	-
19	4.03.1952	42.2	143.9	Tokachi-oki	8.1	0.5	12.8	34	95	58	462	3	-	+	-
20	4.11.1952	52.3	161.0	Kamohatka	8.2	3.5	10.7	138	386	797	18.6	84	-	-	-
21	25.11.1953	34.3	141.8	Boso-oki	7.5	4	3.3	86	156	40	0.6	-	14	-	+
22	22.05.1960	41	73.3	Chili	8.5	4	8.0	209	585	1369	21.4	196	-	-	-
23	26.02.1961	31.6	131.8	Piuganada	7.2	0.2	0.9	22	62	1.7	216	9.4	-	+	-
24	13.10.1963	44.6	149.7	Iturup	7.9	2.5	2.3	105	294	100	5.4	214	-	-	-
25	7.05.1964	40.3	139.0	W.Aomori	6.9	2.3	0.8	32	90	2	0.2	-	8	-	+
26	16.06.1964	38.3	139.2	Niigata	7.4	0.8	3.9	39	81	12.8	25	11.4	-	+	-
27	16.05.1968	40.7	143.6	Tokachi-oki	7.9	1	9.2	63	176	142	142	12.7	-	+	-

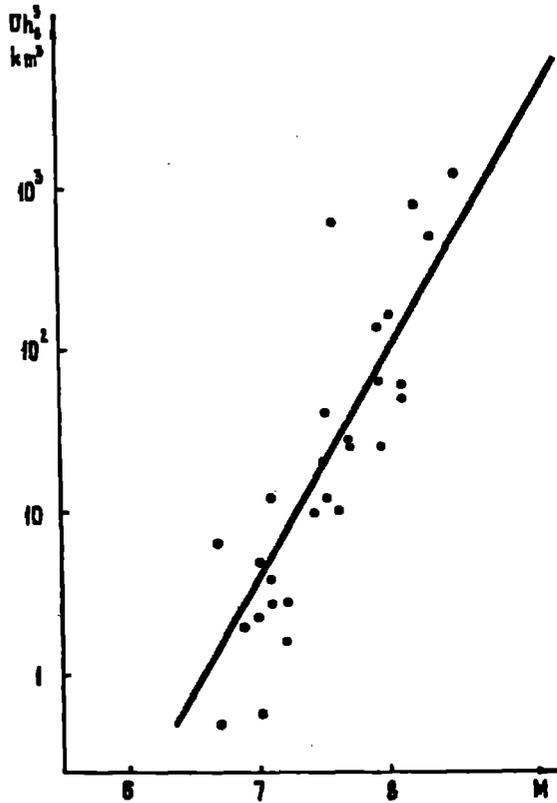


FIG. 1. Correlation field ( $Uh_s^3, M$ ) according to Table 2. Solid line represents regression equation (5).

As seen from Figure 2, the dependence obtained Equ. (6) may serve as a rather good estimate for the Ursell Parameter although its accuracy cannot be estimated because of lack of data on actual tsunamis with a definite ocean depth in the source. It is also seen from Fig. 2 that in the range of magnitude  $6.7 \leq M \leq 8.5$ , the Ursell parameter varies over wide limits for actual tsunamis. Besides, for small ocean depth in the source ( $h < 0.5$  km), the Ursell parameter is rather a large one within the whole range of magnitudes under study, so the nonlinearity proves to be essential for these tsunamis in the open ocean. For larger

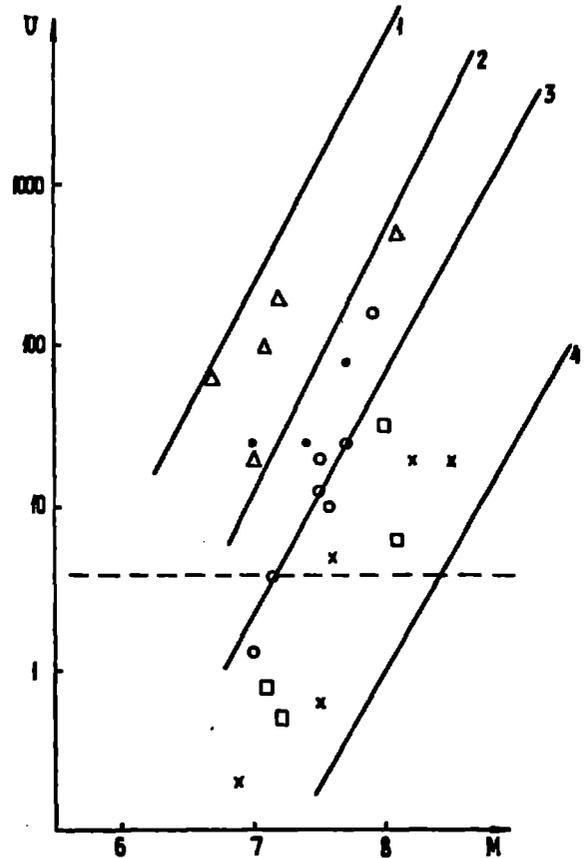


FIG. 2. Relationship between Ursell parameter  $U$  and earthquake magnitude  $M$  for a set of tsunamis as given in Table 2:

- $\Delta$  -  $h_s \lesssim 0.5$  km
- $\circ$  -  $h_s \cong 1.0$  km
- $\cdot$  -  $0.5 \text{ km} < h_s < 1.0$  km
- $\square$  -  $1.0 \text{ km} < h_s \leq 2.0$  km
- $\times$  -  $x-h_s > 2.0$  km

Solid lines represent the relationship (6) when:

- 1 -  $h_s = 0.2$  km    2 -  $h_s = 0.5$  km
- 3 -  $h_s = 1.0$  km    4 -  $h_s = 4.0$  km

Dotted line represents Ursell parameter for a soliton.

ocean depths in the source, the nonlinear effects may be essential only for tsunamis with

sufficiently large earthquake magnitudes. As a rule for actual tsunamis  $U \gg 1$ , however, there are cases when  $U \leq 1$  (approximately 30% of cases). The wave most resembles a soliton in 5 cases [Sanriku, 1896; Sanriku, 1933; Fukushima 6.11.1938; Fukushima, 22.11.1938; Iturup, 13.10.1963], when nonlinearity and dispersion are substantial in the open ocean.

### III. ROLE OF NONLINEARITY

Consider the case  $U \gg 1$  typical of actual tsunamis, when dispersion is not essential over large water areas. Only nonlinearity is important here. We can omit the last term in Equ. (1). The implicit solution for this stage, describing the Riemann wave (a simple one) and remaining continuous during a finite period of time (within a finite length  $L_N$ ) is given in [G. Whitmann, 1974]. This length is a measure of nonlinearity and it also enables one to determine the role of nonlinear effects. The nonlinearity length can be easily obtained [G. Whitman, 1974].

$$L_N = F^{-1}(u_*) - \frac{2}{3} \frac{dF^{-1}}{du}(u_*) \left[ \sqrt{gh} + \frac{3}{2} u_* \right] \quad (9)$$

Here  $F(x)$  describes the particle velocity ( $U$ ) field at the initial moment,  $F^{-1}$  is the inverse function of  $F$ ,  $U_*$  is the water particle velocity, when a wave discontinuity occurs. This velocity is obtained from the equation  $d^2 F^{-1} / dU^2 = 0$ . When calculating the nonlinearity length, a considerable difficulty has been experienced in choosing the initial disturbance form.

Nonlinearity in our approximation is small and we get a simple formula for the nonlinearity length from Equ. (9):

$$L_N = a \frac{\lambda_s h_s}{H_s} \quad (10)$$

where  $a$  is a numerical coefficient dependent on the level displacement form in the source. In particular, if a sinusoidal impulse is given in the source, then  $a = 4/3 \pi \approx 0.42$ . For zero volume of the water expelled in the source  $a = 2/3 \pi \approx 0.21$ . The coefficient is the largest for a symmetrical impulse with linear slopes  $a = 2/3 \pi \approx 0.67$  (the minimum value of  $a$  is naturally zero).

Maximum values of the nonlinearity length calculated from Equ. (10) for a set of tsunamis generated by underwater earthquakes are also given in Table 2. According to these estimates 10 tsunamis of 27 prove to be nonlinear, since the nonlinearity length for them does not exceed the Pacific Ocean extent (see Fig. 3). Nonlinearity can be significant for a larger number of tsunamis if we assume as [M. Brandsma, and D. Divosky and L. S. Hwang, 1978; A. W. Garcia and J. R. Houston, 1979], that the volume of the water expelled in the source is zero. It is still difficult to draw any definite conclusions because of lack of data. We believe that the data cited point to the fact that nonlinearity must be taken into account for a number of tsunamis in the open ocean (for comparatively large distances of thousands and tens of thousands kilometers).

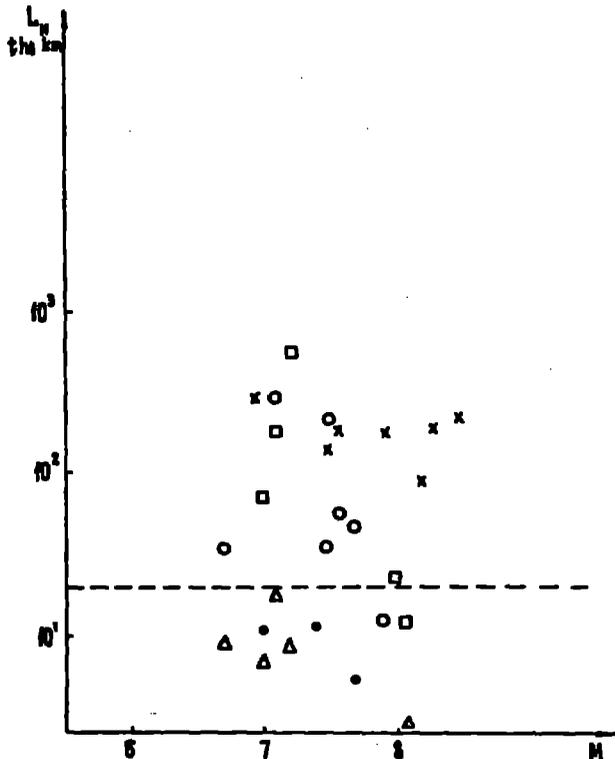


FIG. 3. Nonlinearity length  $L_N^{\max}$  for a number of tsunamis according to Table 2.

- $\Delta$  -  $h_s \approx 0.5$  km
- $\circ$  -  $h_s \approx 1.0$  km
- $\bullet$  -  $0.5$  km  $< h_s < 1.0$  km
- $\square$  -  $1.0$  km  $< h_s \leq 2.0$  km
- $\times$  -  $h > 2.0$  km

Dotted line represents the characteristic extent of the Pacific Ocean.

IV. DISPERSION AND NONLINEARITY

Now let us estimate the role of nonlinearity and dispersion for sources used in schemes of tsunami-zonation. It is assumed in [S. L. Soloviev, et al., 1977] that  $h_s = 2$  m,  $\lambda_s = 90$  km. Then  $U^S > 8$ , if  $h < 1$  km. In another scheme [Atlas of Maximum Tsunami Runup, 1978] it

is assumed that  $H_s = 1-4$  m and  $\lambda_s = 120$  km. Then for  $h < 1$  km we obtain  $U > 7.2$ . In calculation schemes [M. Brandsma, and D. Divosky and L. S. Hwang, 1978; A. W. Garcia and J. R. Houston, 1979],  $U > 16.8$  when  $\lambda < 5$  km. Thus, the Ursell parameter is large in model calculations and therefore, dispersion for tsunami waves can be neglected. This is practically done, since shallow water equations are solved. Besides, when water area is small (for example, for the Kurilo-Kamchatskaya zone), nonlinearity can be neglected too (as a rule the nonlinearity length amounts to some thousand kilometers and longer). This conclusion has been previously drawn on a basis of numerical calculations of linear and non-linear variants [A. S. Alexev et al. 1978; V. G. Bukhteev and A. V. Nekrasov and V. A. Makarov, 1973; V. G. Bukhteev and N. L. Plink, 1978; L. V. Cherkesov, 1976]. We emphasize that these conclusions, however, are valid only for these model examples.

V. ROLE OF DISPERSION

In the other extreme case  $U \ll 1$ , i.e. when dispersion is the predominating factor, it is easy to write down the asymptotic form of the solution of Equ. (1). For distances much longer than the wavelength (but limited), when  $V \neq 0$  ( $V$  is the expelled volume of initial disturbance), we can use the self-similar solution:

$$\eta = \frac{H_s \lambda_s}{2} \sqrt[3]{\frac{2}{h^2 t \sqrt{gh}}} \text{Ai} \left[ \sqrt[3]{\frac{2}{h^2 t \sqrt{gh}}} (x - \sqrt{gh} t) \right] \quad (11)$$

where  $A_i[Z]$  is the Airy function. Another type of the initial disturbance when  $V = 0$  is often considered for tsunami waves. The self-similar solution in this case takes the form:

$$\eta = \frac{H_s \lambda_s^2}{2} \sqrt[3]{\frac{2}{h^2 t \sqrt{gh}}} \frac{\partial}{\partial x} A_i \left[ \sqrt[3]{\frac{2}{h^2 t \sqrt{gh}}} \right] \quad (12)$$

The above exact solutions of the linearized KdV equation (Eqs. (12), (13)) enable one to estimate the dispersion effects in a "pure form". The characteristic distance for dispersion occurrence is given by:

$$L_0 = 0.06 \lambda_s^3 / h^2 \quad (13)$$

(There is an uncertainty in choice of the numerical coefficient in Equ. (13)). We have chosen it from the condition that  $L_D \cong L_N$  for the soliton, since it is evident, that "competitive" effects of nonlinearity and dispersion must appear at equal distances to provide stationarity of wave propagation.

The values of  $L_D$  calculated by Equ. (13) for some actual tsunamis with  $U < 1$  are also listed in Table 2. As seen from these data, finite values of the dispersion length are obtained. If strong elongation of large tsunami sources along the nearest beach is taken into

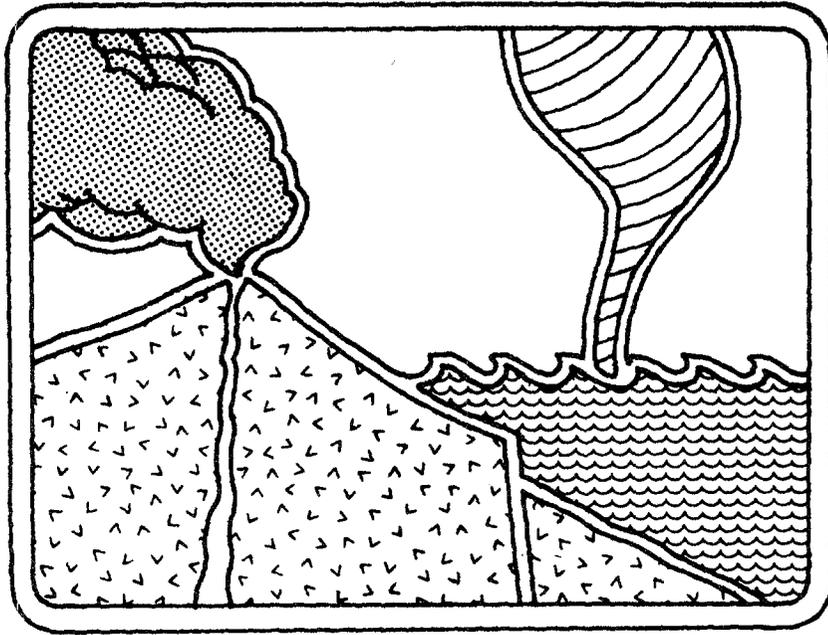
account, then the wavelength determined by the smallest semi-axis may decrease by a factor of 3, and the dispersion length will be significantly smaller than the characteristic dimension of the Pacific Ocean even for tsunami 13.10.1935 Iwate and, hence, the dispersion effects will become essential for these tsunamis. Moreover, the observational data on tsunami wave attenuation cited in [T. Hatori, 1973] for Sanriku 2.03.1933 tsunami are in a good agreement with the  $r^{-1}$  law at distances up to 10 thousand kilometers from the source, that also confirms the dispersion character of wave propagation in this case. The tsunami waves caused by the Aleutian earthquake on 1.04.1946 are dispersive ones, since the wave period increased with distance which is typical of dispersion [S. S. Woyt, 1978].

Thus, the considerations and estimates for actual tsunamis presented here indicate that nonlinearity and dispersion can appreciably affect the tsunami wave propagation at large distances comparable to the extent of the Pacific Ocean.

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## NUMERICAL MODELING OF TSUNAMI FLOODING

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### ABSTRACT

Finite difference programs have been developed for modeling tsunami flooding. Fully centered differences are used. A graphics capability of creating movies showing the results at every time step has been found essential to assure arrival at a believable and correct conclusion. Comparison of output with analytical results, while necessary, does not assure that a program is satisfactory; misuse is not only possible but probable. Most applications documented in the scientific literature allow aliasing in the horizontal interfaces, the vertical interfaces, or both. Smoothing of moving boundary, the shoreline, is necessary to avoid the introduction of spurious high spatial frequencies. It is necessary to see all the computer output--not just the last results.

INTRODUCTION

We wish to be able to predict the conditional expected flooding by a tsunami to within one meter or 30%, whichever is greater. Based on current knowledge, one way to develop this capability is to construct a numerical model for an electronic digital computer. To give a better idea of our objective, we consider the situation shown in the upper portion of Fig. 1.

sions makes this appear much different. At the coastline, there is a rise above sea level, representing a beach dune, then a fall to below sea level, representing a lagoon, rising again to a vertical wall to terminate the model. The method used is that of finite differences; the spatial coordinates, interfaces, incident waves, and time are discretized. The appropriate partial differential equations are approximated by

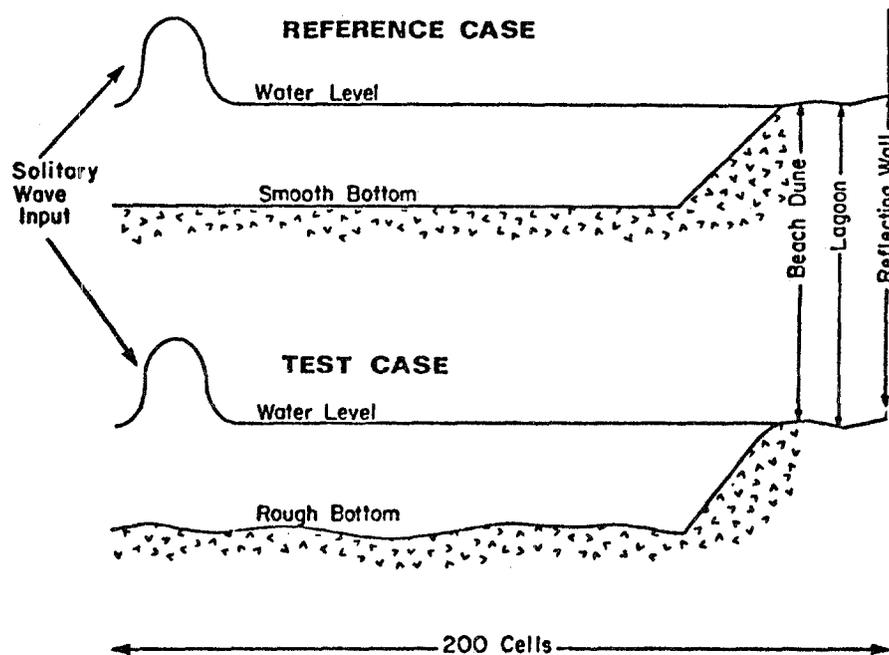


FIG. 1 Cross sections of tsunami flooding models.

This is an idealization of the situation shown in the lower portion of that figure. The cross-section is taken perpendicular to the coastline; and the ocean bottom is taken as perfectly smooth and of constant depth. The bottom shoals gradually to the coastline. A one degree slope is actually used, but the distortion in the vertical to horizontal dimen-

finite differences to the desired order of accuracy and the model programmed for an available computer.

Several computer programs, either finite difference or finite element, have been developed for treating practical situations. Unfortunately each program has been used only by its originator and usually for

only one locality. And most regrettably, not one of those applied programs has been reported as verified. By verified, we mean the comparison of results with the solution of an analytically formulated and solved problem. (In particular, we do not mean calibration, which is the estimation of the values of coefficients by revising parameters until the results fit, sufficiently closely, some observations or expectations.)

Most of the programs developed to date treat the coastline as being a vertical wall, so that flooding is not permitted. In fact, the flooding is usually the information that is specifically desired. Hence, we wish to concentrate upon the case with flooding. This means that we must verify our program by comparison of our results with the solution to some nonlinear equation involving finite amplitudes. Fortunately, two such theories have been published.

We emphasize that using a numerical model which has not

been subjected to verification is extremely hazardous: False predictions may lead to public distrust of hazard predictions in general.

### I. TWO THEORIES AVAILABLE FOR VERIFYING A NUMERICAL MODEL OF FLOODING

One theory suitable for comparison with our computer output is that of [G. F. Carrier and J. P. Greenspan, 1958]. They treat the problem of finding some wave forms that will inundate a uniformly sloping beach without breaking. From the solutions provided, we may choose a test case based upon other criteria, such as those provided by [J. L. Hammack and H. Segur, 1978]. The case which we choose is shown in Fig. 2.

Another theory suitable for comparison with our computer output is provided by the theory developed by [F. K. Ball, 1964] and expanded, with corrections, by [Thacker, 1980]. This is the

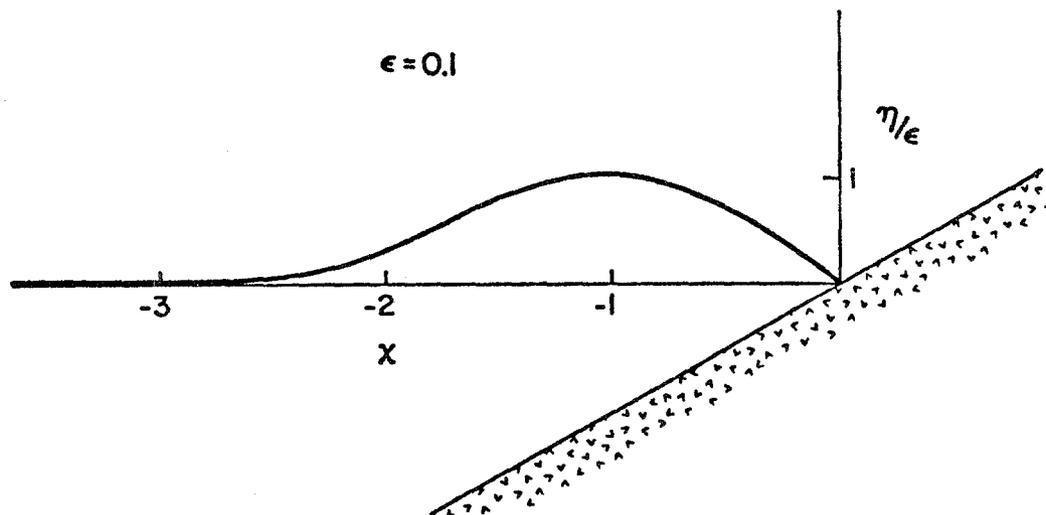


FIG. 2 Case taken from Carrier and Greenspan (1958, Fig. 9).

problem, as formulated by Ball, of finding what bottom topography, if any, can exist that will permit solutions of a specified analytical form. Ball finds that a parabolic basin permits finite amplitude oscillations of a liquid having a time-dependent particle velocity that is independent of position in the fluid mass. The free surface happens to be a plane, and remains a plane (see Fig. 3). Another solution is of an axially symmetric basin having a wave with particle velocity varying linearly with the horizontal dimension (1D or 2D). The surface happens to be a parabola, concave upwards or forwards (see Fig.3). A recent elaboration and correction of this problem's solution is provided by [Thacker, 1980] who takes a somewhat different approach. His inclusion of the Coriolis effect leads to some very interesting oscillations on a parabolic "mountain," the inversion of the basin!

II. FORMULATION OF THE FINITE DIFFERENCE EQUATIONS

The computer program used is that developed for predicting

tsunami flooding. The Navier-Stokes equations are approximated by the usual shallow water equations, vertically integrated.

$$\frac{\partial U}{\partial t} + \frac{\overline{\partial u^2}}{\partial x} = -g(H + \zeta) \frac{\partial \zeta}{\partial x} + \frac{F_B}{\rho}$$

$$\frac{\partial U}{\partial x} + \frac{\partial \zeta}{\partial t} = 0$$

where

$$U = \int_{-h}^{\zeta} u \, dz; \quad \overline{u^2} = \int_{-h}^{\zeta} u^2 \, dz.$$

- g is gravity
- h is water
- F<sub>B</sub> is bottom stress
- ρ is water density

III. FINITE DIFFERENCE EQUATIONS

The difference equations actually used to approximate these equations are evident from the program listing given as an appendix. The program also gives the explanation and details on the procedure for obtaining fully centered values

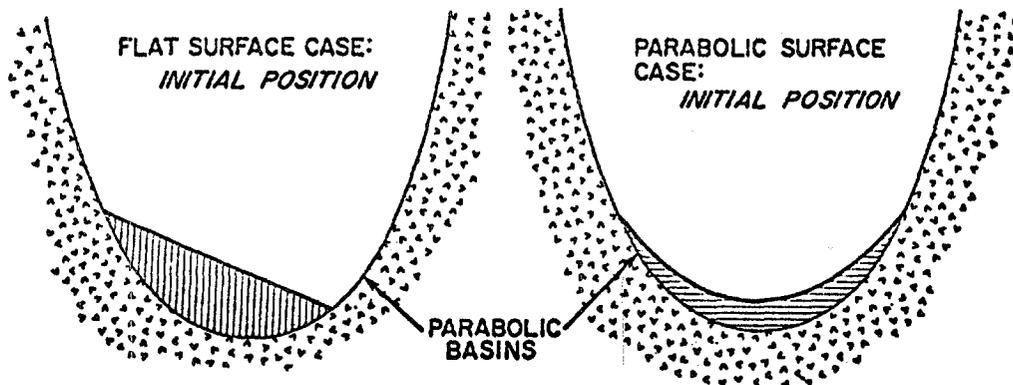


FIG.3 Cross sections of Ball-Thacker models.

by extrapolation and iteration. The most notable improvement over previously published work is the technique for obtaining a fully centered value in time. To do this, an unknown value of the height at a forward time is needed. This is obtained by extrapolation--by fitting a plane through adjacent known values--and then the estimate is used for computing the centered value. Iteration can be used if necessary. However, it was found that the final values for the locations where estimates were made did not differ more than about three percent at most from the planar extrapolation, so iteration was not, in fact, downwards.

#### IV. APPLICATION OF FINITE DIFFERENCE EQUATIONS

It is essential that as much care be taken in applying the finite difference equations as is taken in formulating them. Many of the pitfalls are well known and widely documented. These include stability, truncation effects, and aliasing. The concept of aliasing is often restricted to consideration of the incoming wave. In fact, it must be applied to all interfaces. This becomes apparent, by reciprocity, when the wave enters shallow water and the physics is more current-like than wave like. That is, it does not matter whether the water is moving over the land or the land is moving under the water. Not so obvious is the aliasing arising from a time-dependent interface; for example, the shoreline. This becomes apparent by considering the two-dimension version of the

problem shown in Fig. 1. With a 2-D variable-depth bottom, shown as 1-D in the lower part of Fig. 1, flooding will depend upon position along the coastline. The frequency content of this interface, as generated, has no constraints. Hence, it is necessary to filter or use other techniques to control the aliasing.

#### V. RESULTS

The computation takes place on a 200-cell grid, in the x-direction, and at time steps, for the prototype, of 2.5 seconds. At each cell the height is determined; at each cell boundary, the flux is determined. Thus, for 20 minutes of prototype time, there are 480 time steps, having 192,000 results. Most previous investigators have chosen to concentrate attention on only the final 400 results. We have found it desirable, in fact, necessary, to monitor all the results--intermediate as well as final. The only method that we have found that makes this possible is graphic output of the results in the coherent form of a movie. We have found that this is economically feasible using super-8 equipment and film.

A super-8 movie camera capable of single framing is used with film having a speed of 160 or 400. Color or black and white are possible. The camera may cost as little as \$150.00 and the film ten dollars or less, including processing, in quantity. Almost any digital technician can connect a relay for driving the single frame control

to a parallel port of the digital computer being used.

The results are produced as movie films. Copies of these may be obtained, in limited number, at cost of reproduction (about \$10.00 each) from the author. Here some of the results will be explained by presentation of a few frames from some of the films.

Frames taken from a film on the verification of the numerical modeling computer program versus the Ball-Thacker theoretical results are given in Fig. 4. In frame one is shown the starting configuration. The parabolic basin is shown distorted; the upwards concave surface of the water actually extends over 50 km and is 10 meters deep at the center. In the upper center are the identifying symbols and the time (not visible in this reproduction). At the bottom of the figure is a straight line. This will be used to graph the particle velocity of the water (vertically integrated) in each of the two hundred cells--edge to edge of the figure. By convention, upwards vector at a cell position means particle velocity to the right; downwards vector at a cell position means particle velocity to the left. Truncation of the downward vectors occurs when a threshold value is exceeded, due to the limitation of the CRT screen size. In frame one, the water is just being "released," so the particle velocity is zero everywhere. Notice, in particular, that in spite of the symmetry, the entire problem is being run--not just one half. In frame 2 of Fig. 4 is shown the situation approximately one

minute later. The water on the left is moving to the right; the water on the right is moving to the left. In frame 3, another minute later, the parabolic surface is passing through the special case of its focus at infinity, so it is almost flat. Frames 4, 5, and 6 are about one minute apart as the parabolic surface becomes concave downwards. Frame 5 shows the importance of not running only one half of the problem, arguing from symmetry. Neither boundary is ebbing exactly correct, but the right side is considered better than the left. If only the data of frame 6 had been available, the problem would not even have been noticed! Continuing with frame 7, the flatness of the "deep water" portion shows that section is being handled well, but the moving boundaries are lagging, with the right being slightly more satisfactory than the left.

Frames 8 through 12 show another half-cycle of the oscillation. Now it becomes clear that a program may appear to be correct if only one "cycle" is run. Even though frame 12 has the surface almost flat, significant asymmetry is evident in the particle velocity profile. Thus, monitoring of the surface position is not sufficient. This asymmetry is due to the convention that is normally used, and which was used here, of marching across the cells from left to right. To remedy this fault, marching must alternate; left-to-right, then right-to-left.

Fig. 5 shows frames taken from movies of a solitary-like wave moving from left to right, from deep water onto shore. Vertical

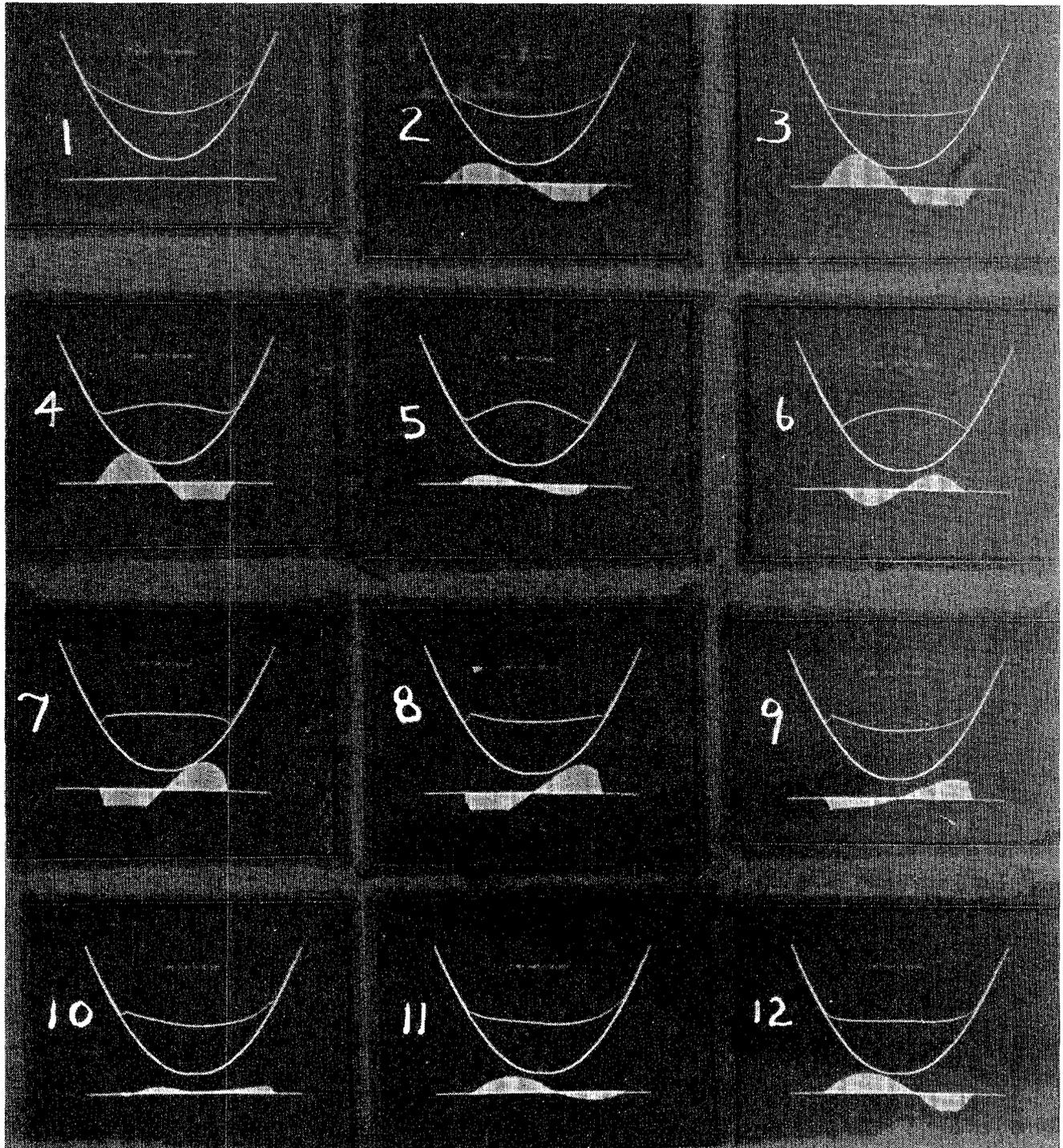


FIG.4 Frames from verification work comparing results with the parabolic-basin oscillations.

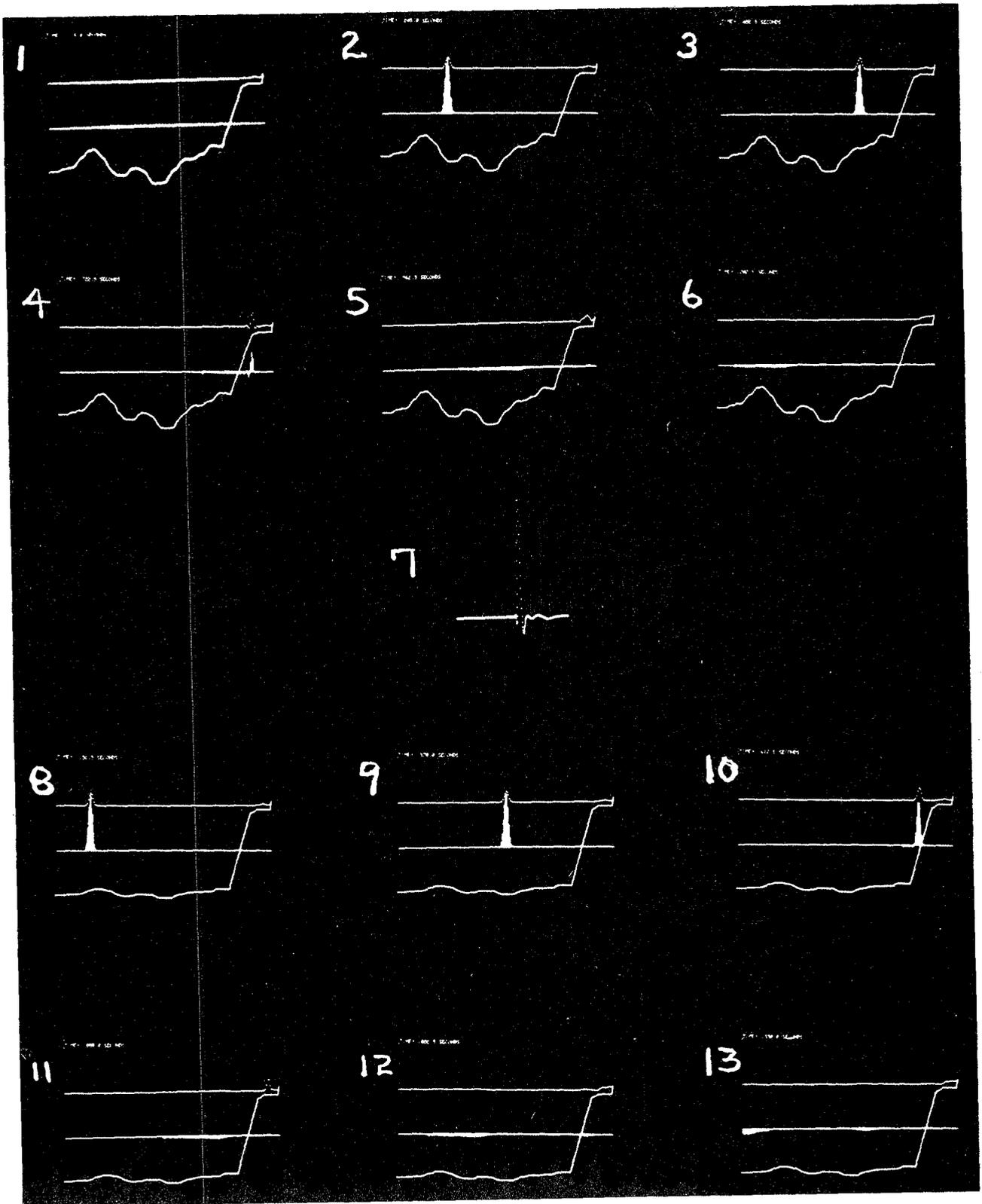


FIG. 5 Frames from study of propagation of gravity wave over a randomly rough bathymetry.

distortion is large, 50. The slope on the right is only one degree! The plot of the surface of the water has been raised slightly to minimize confusion as the wave runs across a shallow lagoon on the right onto dry land. The graph of the particle velocity, shown at the bottom of the frames in Fig. 4 has been raised up into the center of the frame for better viewing. The problem being studied, to be reported in detail in a later paper, is the effect of a rough, variable-depth bottom having specified statistical properties on the propagation of a gravity wave. In Fig. 5, frames 1 through 6 are for one roughness; frames 8 through 13 are for less roughness but the same statistical properties (in fact, the same shape, just a different maximum amplitude). This work is to check the theory such as presented by [Lev A. Chernov, 1960].

Frame 1 shows the starting condition of rough bottom, one-degree slope, lagoon, and beach. In frames 2 and 3, the solitary-like wave is moving from left to right. No reflection is evident. In frame 4, the wave is being reflected from the one-degree slope, as most easily seen on the graph of particle velocity. Frames 5 and 6 show the wave moving very slowly over the lagoon while the reflection is moving relatively rapidly back out to deep water.

Frame 7 shows a graph of the motion of the water surface as a function of time, comparable to what a gage at that point would have recorded. The amplification allows the small complexity introduced by the roughness to be seen. The position of this

(and another) gage recording is assignable.

Another program feature that was found extremely useful in program development, but which is not illustrated here, is the statistics of the number of times a particular loop in the program is actually used. This was very valuable in developing a program comparable to that of [R. O. Reid and B. R. Bodine, 1968].

In summary, these results show the importance, yes, the need for (1) verification, (2) seeing all output, including the intermediate, and (3) the huge requirement for computer time. It must be essentially unlimited.

#### ACKNOWLEDGEMENTS

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- Thacker, (1980).

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C--- CG1-----
C--- PROGRAM TO NUMERICALLY INTEGRATE AIDA'S FORM OF THE LONG WAVE, SHALLOW
C--- WATER NAVIER-STOKES EQUATIONS IN 1-DIMENSION USING EXPLICIT, SPACE- AND
C--- TIME-CENTERED LEAPFROG FINITE DIFFERENCE METHOD
C--- WITH SPATIALLY UNIFORM EVALUATIONS OF APPLICABLE EQUATIONS.
C--- DIAGNOSTIC AND RESTART CAPABILITIES ARE INCLUDED.
C--- A 4TH-ORDER SHAPIRO SMOOTHING OPERATOR IS USED WITH A 1E-6 THRESHOLD.
C--- 3- 5- AND 7-PT. MOVING AVG.'S ARE TAKEN AT THE BOUNDARIES.
C--- ZERO-VALUES ARE UNAFFECTED BY THE SMOOTHING.
C--- A CLOSEUP VIEW OF THE SHORELINE AREA IS DISPLAYED.
C--- A TREATMENT OF  $DQ/DX$  IDENTICAL TO BALL3 IS USED.
C--- Q IS THE VOLUME TRANSPORT PER UNIT WIDTH.
C--- Z IS THE DISTURBANCE HEIGHT RELATIVE TO MSL.
C--- MEANING OF OTHER VARIABLES MAY BE DETERMINED BY EXAMINING PROGRAM.
C-----
C--- INITIALIZE:
C-----
      DOUBLE PRECISION Z(200),Q(200,2),H(200),DDD,HG(200),DELZ
      DOUBLE PRECISION PQIM1N,PQIM1,QTILDE,PZ1N,PZ1,PZ3N,PZ3
      DOUBLE PRECISION ZGRAPH(200),TH(9)
      INTEGER CELL,COUNT(14),FRAMES,CHECKS,CASE(200),ML,WR
      REAL LIN
      COMMON/DFILE/IBUF(4000)
      DATA PI,G,DDD/3.14159,9.8,1D-6/,SWITCH/-1.,/KFIRST/1/,INTRPT/0/
C-----
C--- IDENTIFY RESTART FILES, HYDROGRAPH FILES, INPUT WAVEFORM FILE:
C-----
      CALL ASSIGN(12,'DK1:DATIN.DAT')
      CALL ASSIGN(14,'DK1:DATOUT.DAT')
      CALL ASSIGN(18,'DK1:HX1T.DAT')
      CALL ASSIGN(20,'DK1:HX2T.DAT')
      CALL ASSIGN(22,'DK1:ETAX.DAT')
C-----
C--- TURN CAMERA-ACTIVATING RELAY OFF AND INITIALIZE GRAPHIC DISPLAY:
C-----
      CALL CLEAR
      CALL INIT(IBUF,4000)
C-----
C--- IDENTIFY CELL TO BE INTENSELY MONITORED:
C-----
      CELL=10
      METER=CELL-1
C-----
C--- MORE INITIALIZATION:
C-----
      DO 2 I=1,14
      2   COUNT(I)=0
          PZ1N=0.
          PZ3N=0.
C-----
C--- INPUT GRID PARAMETERS:
C-----
      READ(5,20)N,DX,DT,T,FC,FRAMES,AMP
      20  FORMAT(13,6X,F7.1,3X,F5.1,5X,F8.1,2X,F7.4,3X,12,8X,F7.2)
          READ(5,22)DUMPTM,CHECKS
          22  FORMAT(F7.1,2X,12)
C-----
C--- MORE INITIALIZATION:
C-----
      DO 10 I=1,N
      10  Q(1,1)=0.
          Q(1,2)=0.
          Z(1)=0.
          CASE(1)=0
          CONTINUE

```

```

C-----
C--- READ BATHYMETRY, POSITIVE BELOW MSL, NEGATIVE ABOVE:
C-----
      DO 99 IX=1,N
      99  XI=IX
          HC(IX)=3.-.05*(XI-1.)
C-----
C--- INITIALLY EQUATE WATER LEVEL AND GROUND LEVEL ON DRY LAND:
C-----
      DO 5555 I=1,N
      5555 IF(H(I).LT.0.)Z(I)=-H(I)
C-----
C--- DEFINE INITIAL DISTURBANCE:
C-----
      DO 23 I=1,60
      23  READ(22,5559)Z(60-I+1)
          CONTINUE
C-----
C--- INSURE TOTAL REFLECTION FROM END OF GRID IN ABSENCE OF WALL:
C-----
      H(N+1)=H(N)
      Z(N+1)=Z(N)
C-----
C--- FOR GRAPHICS, USE TRUE SIGN OF ELEVATION RELATIVE TO MSL:
C-----
      DO 5556 I=1,N
      5556 HG(I)=-H(I)
C-----
C--- DETERMINE IF THIS IS A RESTART:
C-----
      READ(5,5557)IRSTRT
      5557 FORMAT(I1)
          IF(IRSTRT.NE.1)GO TO 5558
C-----
C--- REINITIALIZE WITH LAST VALUES:
C-----
      READ(12,5559)((Q(I,J),I=1,N),J=1,2),(Z(I),I=1,N),PZ1N,PZ3N
      5559 FORMAT(D24,17)
          READ(12,5560)SWITCH
          5560 FORMAT(F3,0)
          READ(12,5561)KFIRST,(COUNT(I),I=1,14)
          5561 FORMAT(I7)
          Z(N+1)=Z(N)
C-----
C--- DOCUMENT:
C-----
      5558 WRITE(6,80)DX,N*DX,DT,T
          80  FORMAT('1',' CARRIER-GREENSPAN VERIFICATION, EPS=.1',
          *////,' DELTA X (M): ',F7.1,/, ' RANGE (M): ',F9.1,/,
          *' TIME STEP (SEC): ',F5.1,/, ' TOTAL MODEL TIME (SEC): ',F8.1)
          WRITE(6,840)FC
          840  FORMAT(' ', 'BED FRICTION COEFFICIENT: ',F9.5)
          WRITE(6,842)CELL
          842  FORMAT('0',' FLUX METER READINGS ARE TAKEN AT SEAWARD'
          *,' BOUNDARY OF CELL ',13)
          WRITE(6,82)(H(I),I=1,N)
          82  FORMAT('0',' GRID CELL DEPTHS (M): ',/,/,10(1X,F11.4))
C-----
C--- DETERMINE CURRENT TIME STEP, PRINT CURRENT Z'S:
C-----
      KTIME=0
      IF(KFIRST.NE.1)KTIME=KFIRST
      WRITE(6,83)DT*KTIME
      83  FORMAT('0',/,/, ' T=',F7.1)
          WRITE(6,84)(Z(I),I=1,N)

```

```

84  FORMAT('0',(/,10(1X,E11.4)))
    M=T/DT+1
    IF(KFIRST.NE.1)KFIRST=KFIRST+1
C-----
C----- TIME LOOP:
C-----
    DO 100 K=KFIRST,M
C-----
C----- DETERMINE WHICH COLUMN OF Q IS TO CONTAIN NEW VALUES OF FLUX:
C-----
    IF(SWITCH)86,88,90
88  WRITE(6,888)
888  FORMAT('0','SWITCH ERROR')
    STOP
86  J2=2
    GO TO 92
90  J2=1
92  J1=3-J2
    ZK=K-1
    PQIMIN=Q(1,J2)
C-----
C----- Q LOOP:
C-----
    DO 200 I=1,N-1
C-----
C----- DETERMINE DEPTH AT BOUNDARY BETWEEN CELLS WHERE Q IS TO BE CALCULATED
C----- AND WHERE FLUX METER READINGS ARE TO BE TAKEN:
C-----
    D=(Z(1)+H(1)+Z(I+1)+H(I+1))/2.
    DELZ=Z(1)-Z(I+1)
    DL=(H(METER)+Z(METER)+H(CELL)+Z(CELL))/2.
    ZN=Z(CELL)
    ZN*1=Z(METER)
C-----
C----- UNIFORM FLOODING SCHEME, RECORD-KEEPING, HEURISTIC INTERFACE-
C----- DEPTH ASSIGNMENT:
C-----
175 IF(DABS(H(1)+Z(1)).LT.DDD)GO TO 172
    IF(DABS(H(I+1)+Z(I+1)).LT.DDD)GO TO 174
    IF(H(1).LT.H(I+1))GO TO 176
    IF(Z(1)+H(1+1).GT.DDD)GO TO 178
    D=Z(I+1)+H(I+1)
    DELZ=-D
    COUNT(3)=COUNT(3)+1
    CASE(1)=3
    GO TO 198
178 IF(Z(1).GT.Z(I+1))GO TO 180
    IF(H(1).LT.0..OR.H(I+1).LT.0.)COUNT(4)=COUNT(4)+1
    CASE(1)=4
    GO TO 198
180 IF(H(1).LT.0..OR.H(I+1).LT.0.)COUNT(5)=COUNT(5)+1
    CASE(1)=5
    GO TO 198
176 IF(H(1)+Z(I+1).GT.DDD)GO TO 182
    D=Z(I)+H(I)
    DELZ=D
    COUNT(10)=COUNT(10)+1
    CASE(1)=10
    GO TO 198
182 IF(Z(1).GT.Z(I+1))GO TO 184
    IF(H(1).LT.0..OR.H(I+1).LT.0.)COUNT(11)=COUNT(11)+1
    CASE(1)=11
    GO TO 198
184 IF(H(1).LT.0..OR.H(I+1).LT.0.)COUNT(12)=COUNT(12)+1
    CASE(1)=12
    GO TO 198
174 IF(H(1).LT.H(I+1))GO TO 186
    IF(Z(1)+H(I+1).GT.DDD)GO TO 188
    COUNT(1)=COUNT(1)+1
    CASE(1)=1
    Q(I,J2)=0.
C** GO TO 200
C** GO TO 198
188 DELZ=Z(1)+H(I+1)
    COUNT(2)=COUNT(2)+1
    CASE(1)=2
    GO TO 198
186 D=Z(I)+H(I)
    DELZ=D
    COUNT(9)=COUNT(9)+1
    CASE(1)=9
    GO TO 198
172 IF(DABS(H(I+1)+Z(I+1)).LT.DDD)GO TO 202
    IF(H(I+1).LT.H(I))GO TO 204
    IF(Z(I+1)+H(I).GT.DDD)GO TO 206
    COUNT(14)=COUNT(14)+1
    CASE(1)=14
    Q(I,J2)=0.
C** GO TO 200
C** GO TO 198
206 DELZ=-(Z(I+1)+H(I))
    COUNT(13)=COUNT(13)+1
    CASE(1)=13
    GO TO 198
204 D=Z(I+1)+H(I+1)
    DELZ=-D
    COUNT(6)=COUNT(6)+1
    CASE(1)=6
    GO TO 198
202 IF(H(I+1).LT.H(I))GO TO 208
    COUNT(8)=COUNT(8)+1
    CASE(1)=8
    Q(I,J2)=0.
    GO TO 200
208 COUNT(7)=COUNT(7)+1
    CASE(1)=7
    Q(I,J2)=0.
    GO TO 200
C-----
C----- INITIALLY CALCULATE Q USING ONLY LINEAR TERM; AFTERWARD USE FRICTIONAL AND
C----- ADVECTIVE TERMS:
C-----
198 D=(H(1)+Z(1)+H(I+1)+Z(I+1))/2.
    DELZ=Z(1)-Z(I+1)
    IF(I.GT.1)GO TO 160
    Q(I,J2)=Q(I,J1)+DT*GX*DX
    **DELZ
    IF(DABS(Q(I,J2)).LT.DDD)Q(I,J2)=0.
    GO TO 200
C-----
C----- EXTRAPOLATE NEXT SPATIAL VALUE OF Q TO CALCULATE DG/DX
C----- (NECESSARY FOR PERFECT CENTERING OF ADVECTIVE TERM):
C-----
160 PQIM1=PQIMIN
    PQIMIN=Q(I,J2)
    IF(I.EQ.METER-1)GPROX=Q(I,J1)-PQIM1/2.+Q(I+1,J1)-Q(I+1,J2)/2.
    DDDX=(3.*Q(I+1,J1)-Q(I-1,J1)-Q(I+1,J2)-Q(I-1,J2))/(4.*DX)
C-----
C----- CALCULATE Q NEGLECTING FRICTION:
C-----

```

```

QTILDE=(Q(I,J1)+DT*(GKDDELZ/DX-Q(I,J1)*XGDGX/(2.XD))
&/((1+DT)*DGDGX/(2.XD))
IF(DABS(QTILDE+Q(I,J1)).GE.1E-35)GO TO 162
Q(I,J2)=QTILDE
GO TO 167
C
C--- SEE IF INCLUSION OF FRICTIONAL TERM WILL CAUSE COMPUTER BLOW UP
C
162 QTEST=DT*FCX*(QTILDE+Q(I,J1))*DABS(QTILDE+Q(I,J1))
&/((4.XD)*2*(1+DT)*DGDGX/(2.XD))
IF(.NOT.(QTEST-DDD.LT.QTILDE).AND.(QTILDE.LT.QTEST+DDD))
*GO TO 165
Q(I,J2)=0
GO TO 167
C
C--- SPACE- AND TIME-CENTERED EVALUATION OF Q
C--- WITH LINEAR, FRICTIONAL AND ADVECTIVE COMPONENTS:
C
165 Q(I,J2)=QTILDE-QTEST
IF(DABS(Q(I,J2)).LT.DDD)Q(I,J2)=0.
167 IF(1.EQ.METER)QSAVE=QTILDE
200 CONTINUE
Q(N,J2)=0.
C
C--- END OF Q CALCULATION LOOP;
C--- NOW SMOOTH Q'S:
C
WL=1
I=1
220 ZGRAPH(I)=Q(I,J2)
IF(DABS(Z(I)+H(I)).GT.DDD)GO TO 225
WL=WL+1
I=I+1
GO TO 220
225 ZGRAPH(WL+1)=(Q(WL,J2)+Q(WL+1,J2)+Q(WL+2,J2))/3.
ZGRAPH(WL+2)=(Q(WL,J2)+Q(WL+1,J2)+Q(WL+2,J2)+Q(WL+3,J2)
&+Q(WL+4,J2))/5.
ZGRAPH(WL+3)=(Q(WL,J2)+Q(WL+1,J2)+Q(WL+2,J2)+Q(WL+3,J2)
&+Q(WL+4,J2)+Q(WL+5,J2)+Q(WL+6,J2))/7.
WR=N
I=N
230 ZGRAPH(I)=Q(I,J2)
IF(DABS(Z(I)+H(I)).GT.DDD)GO TO 235
WR=WR-1
I=I-1
GO TO 230
235 ZGRAPH(WR-1)=(Q(WR,J2)+Q(WR-1,J2)+Q(WR-2,J2))/3.
ZGRAPH(WR-2)=(Q(WR,J2)+Q(WR-1,J2)+Q(WR-2,J2)+Q(WR-3,J2)
&+Q(WR-4,J2))/5.
ZGRAPH(WR-3)=(Q(WR,J2)+Q(WR-1,J2)+Q(WR-2,J2)+Q(WR-3,J2)
&+Q(WR-4,J2)+Q(WR-5,J2)+Q(WR-6,J2))/7.
DO 201 I=WL+4,WR-4
DO 2010 J=1,9
TH(J)=0.
IF(I+J-5.LT.1.OR.I+J-5.GT.N)GO TO 2010
TH(J)=Q(I+J-5,J2)
2010 CONTINUE
ZGRAPH(I)=(186.*TH(5)+56.*(TH(4)+TH(6))-28.*(TH(3)
&+TH(7))+8.*(TH(2)+TH(8))-(TH(1)+TH(9)))/256.
IF(DABS(ZGRAPH(I)).LT.DDD)ZGRAPH(I)=0.
IF(Q(I,J2).EQ.0.)ZGRAPH(I)=0.
201 CONTINUE
DO 205 I=1,N
Q(I,J2)=ZGRAPH(I)
205 CONTINUE

```

```

C
C--- CALCULATE NEW DISTURBANCE HEIGHTS:
C
Z(I)=Z(I)-DT/DX*Q(I,J2)
170 DO 300 I=2,N
Z(I)=Z(I)+(DT/DX)*(Q(I-1,J2)-Q(I,J2))
C
C--- PREVENT DISTURBANCE FROM GOING BELOW BOTTOM OR UNDERGROUND:
C
299 IF(Z(I).GE.-H(I)+DDD)GO TO 300
Z(I)=-H(I)
300 CONTINUE
C
C--- ARRANGE FOR TOTAL REFLECTION AT END OF GRID:
C
Z(N+1)=Z(N)
C
C--- SMOOTH Z'S:
C
WL=1
I=1
250 ZGRAPH(I)=Z(I)
IF(DABS(Z(I)+H(I)).GT.DDD)GO TO 255
WL=WL+1
I=I+1
GO TO 250
255 ZGRAPH(WL+1)=(Z(WL)+Z(WL+1)+Z(WL+2))/3.
ZGRAPH(WL+2)=(Z(WL)+Z(WL+1)+Z(WL+2)+Z(WL+3)+Z(WL+4))/5.
ZGRAPH(WL+3)=(Z(WL)+Z(WL+1)+Z(WL+2)+Z(WL+3)+Z(WL+4)
&+Z(WL+5)+Z(WL+6))/7.
WR=N
I=N
260 ZGRAPH(I)=Z(I)
IF(DABS(Z(I)+H(I)).GT.DDD)GO TO 265
WR=WR-1
I=I-1
GO TO 260
265 ZGRAPH(WR-1)=(Z(WR)+Z(WR-1)+Z(WR-2))/3.
ZGRAPH(WR-2)=(Z(WR)+Z(WR-1)+Z(WR-2)+Z(WR-3)
&+Z(WR-4))/5.
ZGRAPH(WR-3)=(Z(WR)+Z(WR-1)+Z(WR-2)+Z(WR-3)+Z(WR-4)
&+Z(WR-5)+Z(WR-6))/7.
DO 301 I=WL+4,WR-4
DO 3010 J=1,9
TH(J)=0.
IF(I+J-5.LT.1.OR.I+J-5.GT.N)GO TO 3010
IF(DABS(Z(I+J-5)+H(I+J-5)).LT.DDD)GO TO 3010
TH(J)=Z(I+J-5)
3010 CONTINUE
305 ZGRAPH(I)=(186.*TH(5)+56.*(TH(4)+TH(6))-28.*(TH(3)+TH(7))
&+8.*(TH(2)+TH(8))-(TH(1)+TH(9)))/256.
IF(DABS(ZGRAPH(I)).LT.DDD)ZGRAPH(I)=0.
301 CONTINUE
DO 304 I=1,N
IF(ZGRAPH(I).LT.-H(I)+DDD)ZGRAPH(I)=-H(I)
IF(Z(I).EQ.-H(I))ZGRAPH(I)=-H(I)
304 Z(I)=ZGRAPH(I)
306 Z(N+1)=Z(N)
C
C--- GRAPHIC OUTPUT:
C
DO 315 I=1,N
ZGRAPH(I)=AMP*KZ(I)
315 CONTINUE
IF(K.EQ.1)GO TO 3301

```

```

CALL INIT
3301 CALL SCAL(39.,-1.1, FLOAT(N+1),2.)
CALL APNT(10.,.5)
CALL TEXT('TIME=')
CALL NMBR(10.,DTXK,'F7.1')
CALL TEXT(' SECONDS')
X1=39.5
IM1=40
IM2=IM1-1
ISTART=IM1+1
C
C--- DISPLAY BATHYMETRY:
C
CALL APNT(X1,HG(IM1))
DO 330 I=ISTART,N
CALL VECT(1.,HG(I)-HG(I-1))
330 CONTINUE
C
C--- DISPLAY FREE SURFACE AND VERTICALLY ORIENTED FLUX VECTORS:
C
DO 331 I=IM1,N
XG=I
IF(DABS(H(I)+Z(I)).LT.DDD)GO TO 370
CALL APNT(XG,ZGRAPH(I))
370 CALL APNT(XG+.4,-.5)
CALL VECT(0.,Q(I,J2))
331 CONTINUE
C
C--- CALCULATE VARIOUS FLOW COMPONENTS AT MONITORED CELL SEAWARD BOUNDARY
C
LIN=DT/DX*GXDX*(ZNM1-ZN)
QNF=Q(METER,J2)+Q(METER,J1)
IF(ABS(QNF).GE.1E-35)GO TO 335
ADVEC=0.
FRIC=0
GO TO 3370
335 FRIC=-DTXFC/(4.*DXLX2)*QNF*ABS(QNF)
337 ADVEC=-DTXQF*(Q(CELL,J2)-Q(METER-1,J2)+Q(CELL,J1)-Q(METER-1,J1))
*/(8.*DXLXDX)
C
C--- REVERSE SWITCH POLARITY:
C
3370 SWITCH=-1.*SWITCH
C
C--- DUMP Z'S, Q'S, AND CASES IF DESIRED:
C
IF(KXDT.LT.DUMPTH.OR.KXDT.GE.DUMPTH+CHECKSXDT)GO TO 3132
WRITE(6,3133)KXDT
3133 FORMAT('0',/,/, ' T=' ,F7.1)
WRITE(6,84)(Z(I),I=1,N)
WRITE(6,3134)(Q(I,J2),I=1,N)
3134 FORMAT('0',/,/,6X,10(1X,E11.4)))
WRITE(6,3135)(CASE(I),I=1,N)
3135 FORMAT('0',/,/,1X,10(10X,12)))
WRITE(6,248)LIN,FRIC,ADVEC
248 FORMAT('0',47X,'FLUX METER:',6X,'LINEAR',4X,'BED FRICTION'
*,2X,'VERTICAL ADVECTION',/,31X,'CHANGE SINCE LAST TIME STEP:'
*,3X,E11.4,2X,E11.4,6X,E11.4)
WRITE(6,1000)QPROX,QSAVE
1000 FORMAT(' ',39X,'APPROXIMATION OF Q:',3X,E11.4,/,52X,' QTILDE:'
*,3X,E11.4)
C
C--- FREEZE FIRST IMAGE UNTIL READY TO PROCEED:
C
3132 IF(K.NE.KFIRST)GO TO 105

```

```

PAUSE
C
C--- ACTIVATE MOVIE CAMERA:
C
105 DO 3136 IFR=1,FRAMES
CALL PHOTO
3136 CONTINUE
C
C--- SAVE HYDROGRAPH READINGS FOR THIS TIME STEP:
C
WRITE(18,3137)Z(40)
WRITE(20,3137)Z(60)
3137 FORMAT(D24.17)
C
C--- IF FRONT PANEL SWITCH 15 IS UP, SAVE VARIABLES AND HALT:
C
CALL CHECK(INTRPT)
IF(INTRPT.NE.1)GO TO 100
WRITE(14,5559)((Q(I,J),I=1,N),J=1,2),(Z(I),I=1,N),PZ1N,PZ2N
WRITE(14,5560)SWITCH
WRITE(14,5561)K,(COUNT(I),I=1,14)
END FILE 14
STOP
100 CONTINUE
C
C--- END OF TIME LOOP;
C--- OUTPUT STATISTICAL COMPILATION OF ADJACENT CELL WATER LEVEL CONFIGURATIONS
C--- INVOLVING AT LEAST ONE DRY CELL:
C
WRITE(6,102)
102 FORMAT('0', ' CASE COUNT')
DO 103 I=1,14
WRITE(6,104)I,COUNT(I)
104 FORMAT(' ',2X,12,4X,17)
103 CONTINUE
STOP
END

```

**CYLINDRICAL SOLITONS  
PASSING THROUGH A FOCUS**

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## INTRODUCTION

Many theoretical and experimental investigations have been performed on the propagation of nonlinear water waves particularly in connection with tsunami waves. The evolution of the solitary quasi-stationary waves - solitons is the subject of special interest. The propagation of cylindrical solitons has been considered in many papers. Experimental investigations of the solitons on water [Tsukabayashi, I. and T. Yagishita, 1979], in plasma [Hershkowitz, N. and T. Romesser, 1974; Chen T. and L. Schott, 1977; Nishida, T. and T. Nagasawa and S. Kawamata, 1979] and in two-dimensional electric lattices [Stepanyants, Yu. A., 1982] have shown some differences of the laws of wave amplitude variations from predictions of a linear theory and the considerable transformation of the wave field within the focal region. To obtain more reliable qualitative relationships, governed by the axisymmetric Korteweg-de Vries (KdV) equation or more exact equations, special numerical calculations were performed [Maxon, S. and J. Viecelli, 1974; Ogino, T. and S. Takeda, 1976]. The calculations corresponding to adiabatic conditions (slow variation of the soliton parameters) are adequately described by the approximate theory of cylindrical solitons [Ko, K. and H. H. Kuehl, 1979].

### I. THE MATHEMATICAL PROBLEM

In particular, the amplitude of a soliton moving to the cen-

trum increases, according to the formula  $A \sim r^{-2/3}$  ( $r$  is the distance to the centrum) and, the duration decreases as  $T \sim r^{1/3}$ .

Since the KdV equation is invalid near the focus, the results available on focus transformation of cylindrical solitons still remain to be interpreted. This problem is largely close to the problem of transformation of the intense acoustic waves in near-caustic regions and for complete internal reflection at the interface [Ostrovsky, L. A. and E. N. Pelinovsky and V. E. Friedman, 1976]. The basic idea for obtaining of analytical results used in [Ostrovsky, L. A. and E. N. Pelinovsky and V. E. Friedman, 1976] is associated with the possibility of passing through the near-caustic regions according to the linear theory formulas. In fact, even in nonlinear geometrical acoustics an infinite increase of the wave amplitude on caustic is not accompanied by a significant increase of nonlinearity (the ratio of harmonic amplitudes to the fundamental wave amplitude remains finite). On the other hand, the influence of diffraction in this region manifesting itself as phase change of each harmonic by  $\pi$  becomes predominant. The same situation must certainly take place in dispersive media: in the regions with dimensions much smaller than the characteristic lengths of nonlinearity and dispersion, the linear approximation formulas can be used. In this case the relationship between the incident and transformed wave is given by the Hilbert transform [Ostrovsky, L. A. and E. N. Pelinovsky and V. E. Friedman, 1976]:

$$\psi_{ref}(t, z_0) = \frac{1}{\pi} \int_{-\infty}^{\infty} \psi_{in}(t', z_0) \frac{dt'}{t-t'} \quad (1)$$

where  $\int_{-\infty}^{\infty}$  represents the principal-value integral. Fig. 1 displays the transformed wave field (solid line), when the incident wave is the KdV soliton (dotted line)

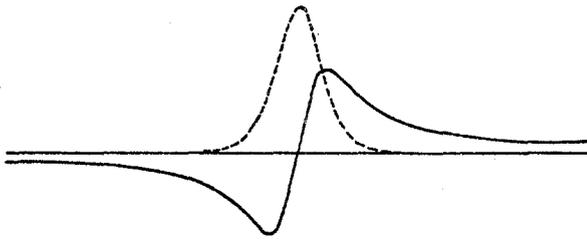


FIG. 1. Hilbert transform (solid curve) of the KdV soliton (dotted curve).

$$\psi_{in}(z_0, t) = A \operatorname{sech}^2 \frac{z_0 - V_0 t}{\Delta} \quad (2)$$

## II. DISCUSSION OF RESULTS

As seen from Figs. 1 and 2, the form of the transformed wave has considerably changed - the

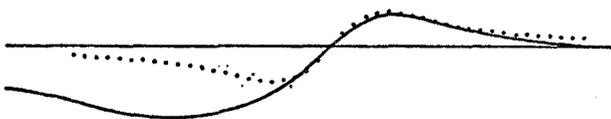


FIG. 2. The transformed wave after focus passing of KdV soliton. Solid line - computation according to exact equations [A. T. Chwang and T. Y. Wu, 1977; T. Y. Wu, 1979], points - calculations by formula (1).

wave has become an alternation and a significantly wider one. One can show from Equ. (1) that the transformed pulse field decreases in its asymptotic form as  $\sim |t|^{-1}$ . However, this result is an approximate one in its physical sense, since the main contribution to the pulse tails are made by lf components of the wave spectrum ( $\lambda \gg r_0$ ) for which the transformation Equ. (1) is invalid (the focal region for such components is large and at  $\omega \rightarrow 0$  it approaches infinity). Most detailed transformation of the KdV soliton in the focus have been computed by [Hwang, A. T. and T. Y. Wu, 1977; Wu, T. Y. 1979] by solving exact hydrodynamic equations. Fig. 2 exhibits the profile of the wave having passed through the focus. Dots in this figure show the results of calculation by Equ. (1). Good coincidence of the data obtained by Equ. (1) and the numerical calculation [Hwang, A. T. and T. Y. Wu, 1977; Wu, T. Y. 1979] is observed for the head part of the wave that departed well enough from the center. The difference in the tail is caused by its proximity to the center, so that diffraction has not manifested itself completely.

The transformed wave propagating away from the center is evidently not stationary and when evolving, it breaks into solitons and oscillating trains. To calculate this effect, we neglect the cylindrical divergence, meaning the estimation of the upper limit on parameters of secondary solitons (this gives exact values for soliton amplitudes in problems on complete internal reflection of a plane wave at the interface). Breaking into solitons is due to the

positive head part of the reflected pulse, which, as shown above, is well described by Equ. (1). In the framework of the KdV equation the number of secondary solitons from the wave reflected from the focus is equal to infinity, since the Schrödinger operator associated with this equation [Karpman, V. I. 1973] possesses an infinite number of energy levels accumulating to zero for such slowly decreasing potentials [Glazman, I. M. 1963]. The first soliton amplitude (normalized to the amplitude of the initial soliton converging to the focus) is equal to 0.51, the second soliton amplitude equals 0.085, etc. Since a slow decrease in asymptotic forms of the transformed wave is caused by insufficient proximity, then only some first solitons are physically real, and only one-two solitons are observable due to a rapid decrease of their amplitudes.

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## ESTABLISHMENT AND OPERATION OF A TSUNAMI MONITORING PROGRAM

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### ABSTRACT

An extensive program for real-time and post-event monitoring of tsunamis has been developed and implemented for the Hawaii area. Methodology includes improved deployable instruments, stop-motion photography from many selected locations, real-time photography from the air, and post-event surveys from the ground and air. Use of existing facilities and organizations, and how to cooperate with and coordinate them is described, and equipment is stressed. Application of these techniques to other locales is outlined.

## INTRODUCTION

A tsunami monitoring program is in progress to organize and plan for significant real-time and post-event measurements of tsunamis, before the need arises. Various disciplines and organizations perceive different, often overlapping reasons for providing such data.

These include:

### Researchers

- travel time predictions
- magnitude predictions
- run-up predictions
- diffraction effects

### Civil Defense

- time of arrival predictions
- run-up predictions
- inundation predictions
- arrival direction effects
- refinement of warning systems

### Planners

- coastal zone "management"
- insurance estimate
- risk assessment
- property value assessment

Essentially, we are discussing a data collection and evaluation project for which there is limited time to collect data and ample time to analyze them. Since a monitoring program is primarily concerned with collecting (or preparing to collect) data it is conveniently divided into two time-related modes: real-time and post-event. The real-time temporal limitations are obvious; in the post-event mode these are still present but are much slower paced, and with an exponential decay. Both of these modes are used to collect data for the

needs outlined above and will be discussed in order. The Joint Institute for Marine and Atmospheric Research (JIMAR) at the University of Hawaii conducts such a program in cooperative with NOAA and the State of Hawaii.

## I REAL TIME METHODS

### A. Time lapse photography

Our primary dependence is a time lapse photography. Initially, some surplus 16 mm gunsight cameras were modified with intervalometers and battery packs. These have been superseded by commercial Super-8 movie cameras with auxiliary intervalometers and more recently by a simple and relatively cheap (cost and construction) surveillance camera with a built-in intervalometer.

These cameras are issued to volunteer observers who are trained briefly in their use and agree to (1) keep camera, film, and batteries properly stored and at hand; (2) proceed to a designated or alternate site upon a warning or notification; and (3) test themselves and their equipment by shooting a roll of film annually. Their check/procedures list is shown in Fig. 1. The observers are encouraged to have their film processed, personally viewed, and then sent in to us for further review. Sending a roll of film and a development mailer is a helpful reminder. Film is usually exposed at 1 frame per second; normal speed projection takes approximately 4 minutes for a 16 to 1 speed up.

TSUNAMI CAMERA OPERATOR/OBSERVER  
CHECK AND ACTION LIST

1. Post this list in a convenient place and keep a copy with the equipment.
2. Keep track of status and ETA of event by radio, and telephone if necessary. (See number below.)
3. Remove any refrigerated materials early (film, batteries).
4. Prepare 9 AA cells and 1 cartridge of super 8 film for loading. Select film type for ETA. Set out 5 spare AA cells and 3 cartridges film. Select film type for post-arrival light conditions. Prepare appropriate still film rolls. Mark film A, B, C, D, etc.
5. Load cameras and check for normal operation (including exposure meter functioning) and insure your familiarity with them. (Note: film and batteries should be free of condensed moisture before loading.)
6. Check \*and pack equipment: Movie camera \_\_\_\_\_, intervalometer \_\_\_\_\_, trigger switch \_\_\_\_\_, still camera \_\_\_\_\_, tripod(s) \_\_\_\_\_, film \_\_\_\_\_, batteries \_\_\_\_\_, radio \_\_\_\_\_, pen \_\_\_\_\_, notebook \_\_\_\_\_, CD card \_\_\_\_\_, watch \_\_\_\_\_, kitchen timer \_\_\_\_\_, key to location \_\_\_\_\_, other \_\_\_\_\_.
7. Call regarding intended/desired location; phone 948-8083. Set watch by HTCO (543-3211 or WWVH (471-6363).
8. Pick any necessary spare film and batteries enroute.
9. Upon arrival, set up cameras and check operation. Call in and report phone number at location.
10. Ten minutes before ETA start movie camera @ 1 frame per second (FPS). Try to include a permanent reference point in viewfinder to re-orient camera if jarred, or moved on purpose. Set kitchen timer for 55 minutes (duration of standard cartridge is one hour at 1 FPS). Start notebook.
11. Concentrate stills on (i) reference points (for water level) and shore area conditions before wave, (ii) minimum and maximum wave heights and water lines, (iii) reference points and shore area conditions after waves and (iv) reference points of known size for scale.
12. Record times and still frame numbers (approx.) in notebook. Make liberal notes (focal length of lens, visual observations, etc.)
13. Change super 8 cartridge at convenient time before it runs out; reset timer. Mark cartridge if not already done and enter time in notebook.
14. Call for runner for more supplies, if needed, in ample time.
15. Review the above, and double check your actions.
16. Additional notes: \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_
17. Phone numbers: Honolulu Observatory-PTWC 689-8207, \*; JIMAR 948-8083, 948-8084; Birkheimer 734-2151, \*; Loomis, 373-6153; Adams, 955-0848; Vitousek, 737-0202; Curtis, 239-7263; ITIC, 546-2847.  
\* Unlisted number - do not disclose

FIG. 1. Tsunami Camera/Observer Check and Actions List. (Form)

Site selection for both ground and aerial photos depends on an analysis of several factors, including historical data and predictions, as reported in the literature [G. D. Curtis, 1980]. While the aerial portion was detailed with the aid of experienced pilots and involves air-space considerations, the ground observers' stations involve several mundane but important factors and compromises, including the following:

- location of suitable personnel;
- transportation, especially during an evacuation;
- storage of cameras, film and batteries;
- total travel time of observer to site;
- safety, security and weather protection at site;

- Civil Defense clearance to remain in evacuation area;
- elevation and sight lines.

To test the above and provide baseline data, still photos are made at each site, checked, and filed.

Fig. 2 shows present locations in the Hawaiian Islands. A number indicates multiple cameras and/or observers are available to optimize coverage. For example, the Hilo area is covered from two locations, each in upper stories of shoreline buildings with two cameras aimed in opposite directions. If only one observer happens to be available, he can set the cameras up with, say, a 4 second interval and actively man only one site.

Although it appears photography would only work in daylight, observers will be deployed if one wave can be

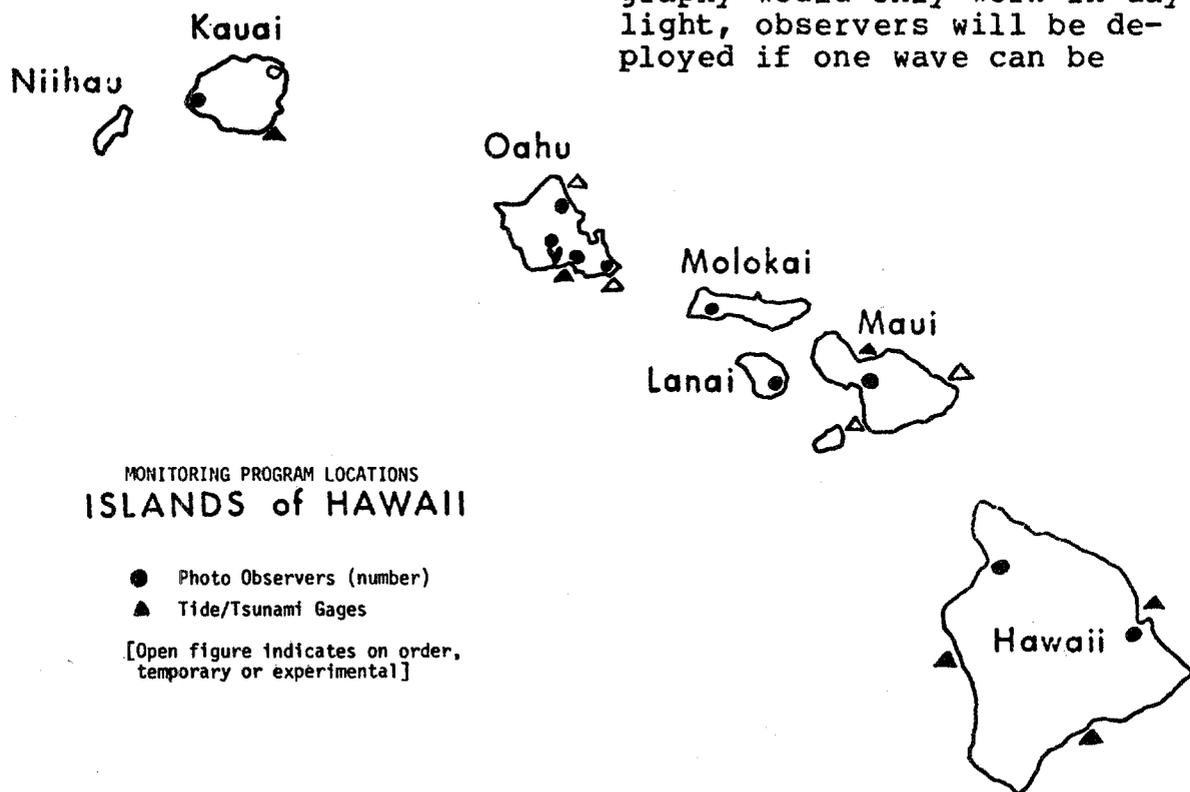


FIG. 2. Monitoring Program Locations. (Chart)

recorded before dark or if there is a possibility of later waves (and prior wave effects) being photographed after dawn. In two harbor locations, limited night time photography is feasible, using lighted floats and the stern lights of evacuating boats.

Two of our observers are dedicated photographers who provide their own cameras. And, all provide still cameras to ensure adequate coverage of salient wave action and reference points. We provide them with Civil Defense Special Area Passes to facilitate access to their sites during a tsunami alert.

### B. Aerial photography

We have established a novel project to make aerial photos of a tsunami running up on a coastline. It is based on the presence and cooperation of Naval patrol aircraft. These are P-3B's, with remote cameras in the bellies which can take 480 exposures on 70mm film, on co-pilot command or with an

intervalometer. The pilot flies a racetrack pattern about 10 miles long in a selected area; a sequence of pictures of the shoreline area is taken about every 15 minutes. An example is shown as Fig. 3. The runs are made at 1000 ft.; there is some overlap depending on intervalometer inaccuracy, but stereo viewing has not shown measurable relief in storm waves. This appears to be a good altitude for the speed and camera involved. (The camera is marginal for its intended purpose of photographing ships, etc., but is satisfactory for coastal photography.)

The Navy squadrons involved are provided with a briefing kit which includes details not only on how to fly the mission, but also on how to select best areas based on tsunami origin, E.T.A., and cloud-cover alternates. Thus, if we are unable to contact them and provide an observer, they can proceed anyway. The photo areas selected are based on a number of factors which have been analyzed and reported in [G. D. Curtis, 1980]. This report is also one

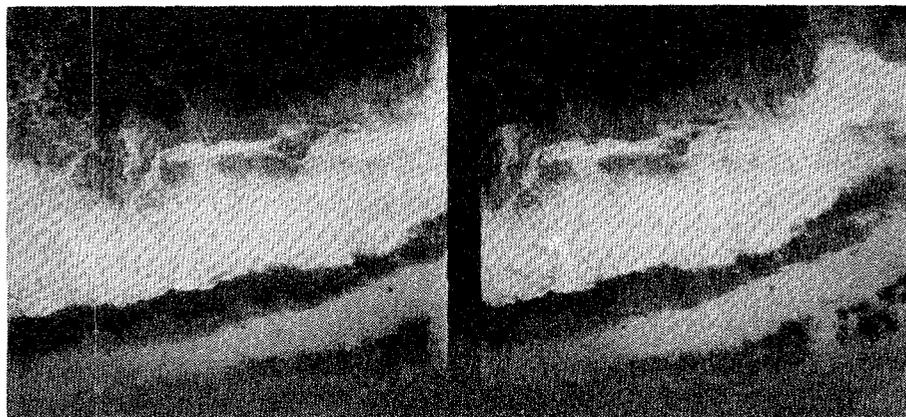


FIG. 3. Photo Sequence from P-3 at 1000 ft.

of the bases for determining time-lapse photo sites; a similar matrix could be developed for any tsunami-prone coastal area.

Low-level oblique photos of areas of special interest during a tsunami will be provided (on a non-priority basis) with the aid of the Civil Air Patrol. In Hawaii their standard tsunami procedure in support of the Civil Defense Agency is to overfly coastal areas of several islands with loudspeaker-equipped aircraft and warn people to evacuate. After circling the island, the aircraft is to land, refuel, and commence a patrol to check problem spots, assess damage, etc. During this phase, they can carry one of our observers who will take sequences of still photos from 500 ft. The Civil Air Patrol's knowledge of the area and timing of the waves, and its ability to observe and to change locations rapidly, proves to be a very useful resource.

Another thrust of the monitoring program is the development and deployment of tsunami gages. Our current analog-strip chart type (Fig. 4) is based on a solid-state strain-gage pressure transducer and incorporates the battery, amplifier, filter, and chart recorder in one unit. Pressure (water level) is sensed at the end of a hundred feet of plastic tube. These devices are pre-located in the homes of observers who live at or near a suitable site. Upon warning such observers will turn the gages on, deploy the tubes to an anchor point in several feet of water, set the instruments for the depth and expected range,

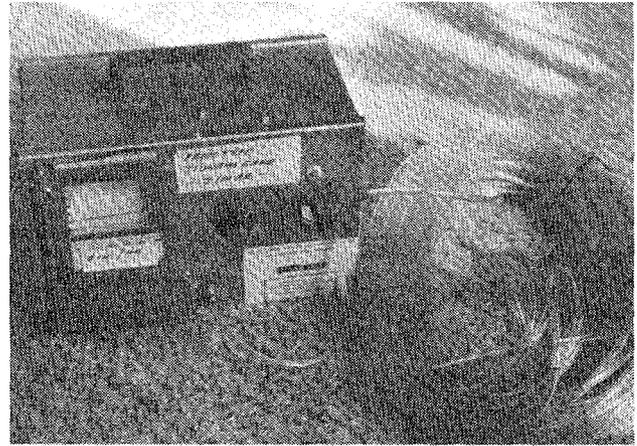


FIG. 4. Analog Gage.

and leave them while proceeding to perform their other tasks such as photo-station.

A more sophisticated digital gage is presently under test. Fig. 5 shows its principles.

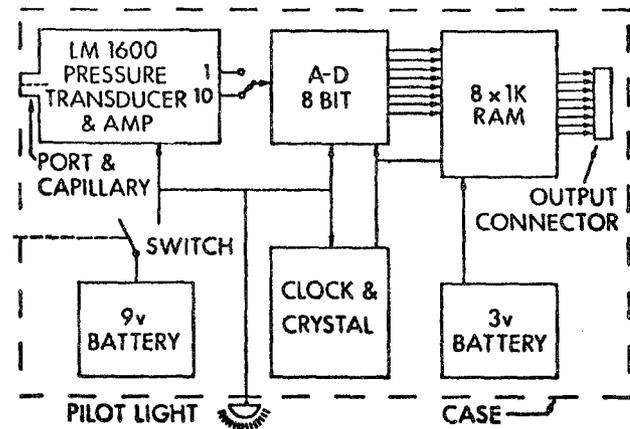


FIG. 5. Digital Gage.

The transducer is the same as that used in the older analog model but has more hydraulic and electronic low pass filters to prevent aliasing; after digitization of a sample twice a minute, the eight-bit data word (which has a resolution of better than 0.5%) is stored in a

semiconductor memory. While one prototype was made using a volatile memory (RAM), the present model sets each data sample permanently into an electronically programmable, read-only memory (EPROM). The numbers can later be read out in the lab and displayed on a strip chart, or "listened to" with sound that varies frequency for pressure variations, and "seen" with graphics on a CRT that show variation patterns in the data.

Using this gage is relatively simple. The device is loaded with fresh batteries upon receipt of a warning, taken to a nearby prepared site, turned on, and mounted underwater or connected to a tube leading into the water. The instrument is watertight, uses common batteries, records for 12 hours, and is relatively inexpensive to build and maintain. The gage is not limited to a onetime use; installing a new \$2.00 EPROM can make it ready for use again. There are no moving parts (ex-

cept the switch); it is not affected by overpressure; and as long as it can be retrieved after an event, it should provide much usable data. The normal range is 10 meters but a 2 meter range is included to encourage its deployment in the event of a small tsunami or in protected locations. As [National Science Foundation and National Oceanic and Atmospheric Administration, 1981] states, suitable gages are a primary need and have high priority for tsunami R & D. This gage is intended to fill a part of this need by providing a number of recording systems on long-term standby in many locations. A photo of the final configuration is shown in Fig. 6.

Of course, tide gages operated by N.W.S. and others are a part of any monitoring program and provide important long-term reference data. Unfortunately many are located in harbors and subject to significant spectral contamination. In addition,

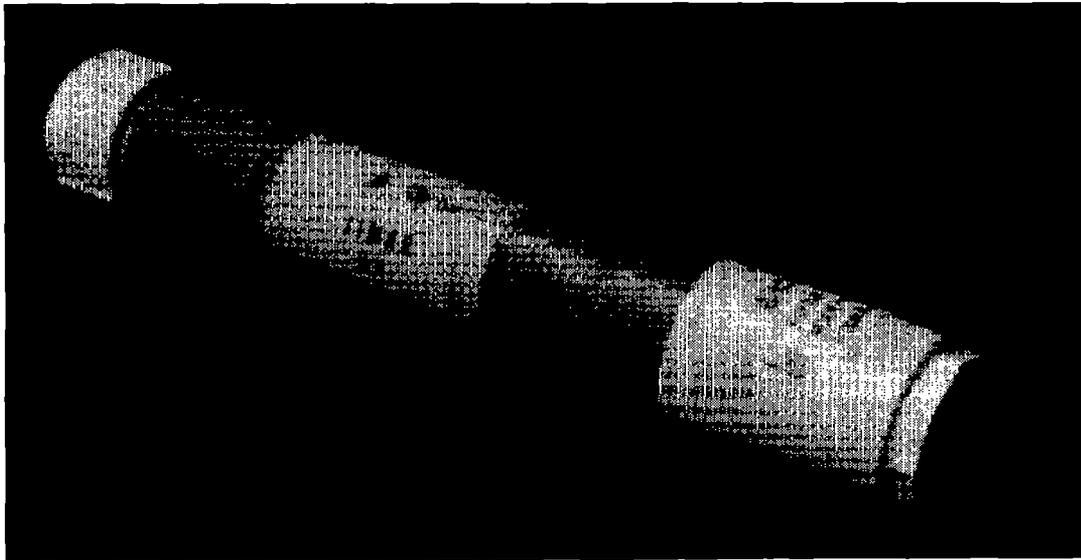


FIG. 6. Digital gage prototype.

their frequency responses (stilling well characteristics) are often not well known. Gages optimized for tsunami have been developed by Van Dorn and others, but the installations have regrettably not been maintained. We are fortunate in Hawaii to have three telemetered (remote) gages in clear locations operated by the Pacific Tsunami Warning Center (PTWC) and two others of differing characteristics operated for an unrelated purpose by the Hawaii Institute of Geophysics (HIG, U.H.). Knowledge of and contact with such resources are an important part of a tsunami monitoring program. (Fig. 7 shows a small tsunami recorded by one of the geophysical gages.)

An unusual resource which may be added to our monitoring capability is a sort of "inverted fathometer" found by recording signals from pairs of active and passive hydrophones at a Naval

range just offshore of the island of Kauai. Our initial trial - after cleaning up most of the "noise" from swells - did not show the tide (range approximately 2 ft.) as we had hoped, but an unexplained, shorter period water level change. If there is a tsunami warning, however, the system will be activated and a recording made for later analysis.

Clearly, it is our intent to get real-time data by every means possible because of the value of the added dimension of time.

## II. POST-EVENT METHODS

The lesser time pressure after a tsunami does not lessen the need for adequate preparation, planning and prompt action. Our post-event data are gathered primarily by photography and ground surveys, and we have made advanced efforts in both areas.

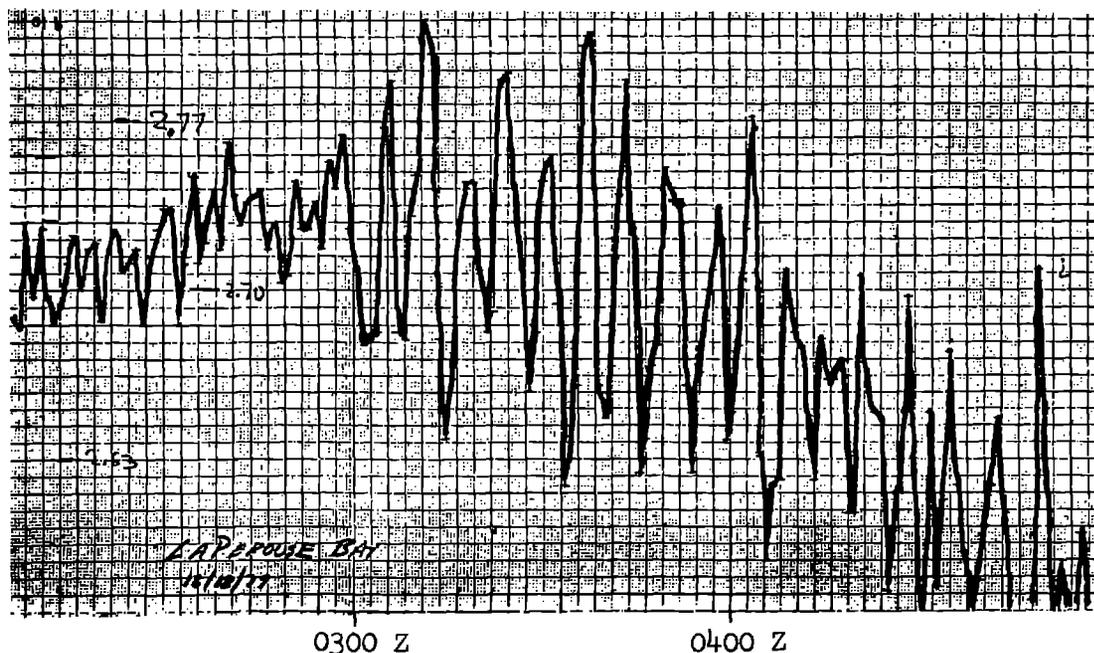


FIG. 7. Mini-Tsunami of Dec. 12, 1979, La Perouse Bay, Maui.

These emphasize the need to gather the evidence before it disappears and while interest, people, and funds are available.

#### A. Air surveys

The photo effort starts with identification of baseline photo sources. Contacts have been established with both the commercial photogrammetric operators and the Federal and State agencies who are their main customers. Photos on file have been spot checked, with some emphasis on locating color pictures because these are more revealing. Color photos with fairly good resolution are available from U-2 operations; Landsat and other video satellite pictures are inadequate. Cooperation from various agencies has yielded generally good coverage of the coastline in the last decade.

After an event, we will again get quick response photos of priority areas via the Navy and Civil Air Patrol. We will set priorities at the time and try to send an observer along so that further selection can be made in the air. Oblique color photos taken after the local 1975 tsunami on the Island of Hawaii showed good delineation of high water marks in many inaccessible areas. We would expect to repeat such photo runs for specific details and additional areas as effect patterns emerge. An informal arrangement with the Coast Guard will allow photography and observation on a non-priority basis in selected areas.

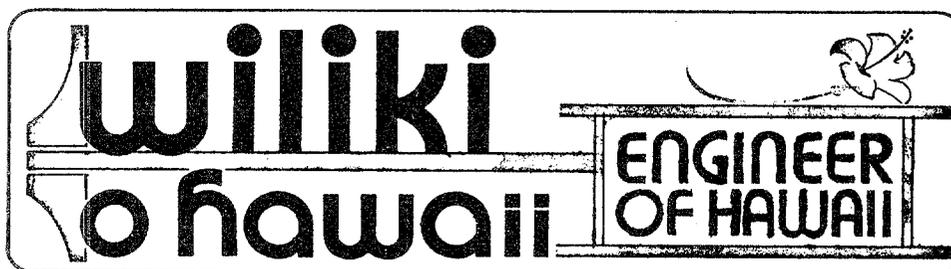
As soon as areas of greatest interest have been agreed upon,

we plan to hire a commercial aerial photo company to cover those areas. With a 24-hour advance notice, color can be provided. This approach will yield large, high resolution pictures of known scale for detail analyses.

We will request, through NOAA, photos by the U-2's operated by the National Weather Service. Since these flights are planned well in advance and there is no loiter time, cloud cover can be a problem; in this case, we will ask for a repeat. We will also request through NOAA and other channels that photos be made by Air Force reconnaissance satellites (starting as soon as a damaging tsunami is known to be in progress). Although security may prevent the photo's immediate release, they should become useful records for future use.

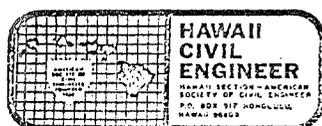
#### B. Ground surveys

For ground survey purposes plans have been made with a volunteer group from the American Society of Civil Engineers (see Fig. 8). The state chapter has accepted this as an on-going project. Personnel and resources (maps, four-wheel drive vehicles, etc.) have been listed for each major island. In fact, a complete set of quadrangle charts for the coastal area has been placed in an ASCE member's office on the island for which he has agreed to act as coordinator. A great advantage is that these people already have local knowledge, experience, and contacts. Joined with us in these arrangements is the Corps. of Engineers who have a disaster-survey team always available. Although their primary task is



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### Request to Assist in Tsunami Research

The Hawaii Section, ASCE, has been asked by the Joint Institute for Marine and Atmospheric Research to provide technical assistance for follow-up studies of tsunami activity. The work requested will be to map the high water mark and damage limits following an inundation. Since tsunamis are infrequent events, a long-term commitment by the volunteers is required.

The program will require approximately 24 engineer volunteers throughout the State. One coordinator will be appointed on each island. His responsibility will be to maintain an up-to-date list of people and to arrange their efforts after a tsunami. Training will be provided and materials and information distributed.

Following a tsunami, the island coordinator will contact the engineers on his island and assign areas of work. Engineers will be given passes into the area and will mark and map the extent of high water. This information will then be used by the Institute to determine future plans for disaster avoidance.

If you would be willing to give some of your time and skills after a tsunami disaster, please call Richard Fewell at 836-2171 for further details. This is an excellent chance for us to be of assistance to our communities.

FIG. 8. ASCE Notice.

damage assessment, the surveys will be mutually supportive. We feel that feedback from the aerial surveys (the hand camera photos are available overnight; the Navy vertical photos in about 24 hours; and personal observations immediately by telephone or radio) will be particularly useful in deciding where to concentrate the ground surveys. A manual of procedures for conducting such surveys is now in its final editing process and will be distributed to all personnel in 1982. It draws on material such as [State of Hawaii, 1981] and prior publica-

tions, as well as direct experience.

### III. OBSERVERS AND ORGANIZATIONS

The previous paragraphs reveal the general approach we have developed locally to handle the tsunami monitoring program. Many of these precepts will certainly apply elsewhere; e.g., most of the people involved will be functioning as "volunteers" even if they are paid by some agency in another capacity. This must be understood and ade-

quate support, feedback, and recognition provided. It is akin to maintaining a volunteer fire department (most of whose members have never ever seen a fire) with a part-time chief amid an assortment of government officials who have primary interest. The solution is to recognize these factors and work with them, through them, and occasionally around them. The most vital factor is to show the monitoring team that they are fulfilling an important need. This applies to both organizations and individuals. We are fortunate to have an experienced, in-house nucleus of people at the University of Hawaii with long experience in tsunami work. Though most are no longer directly involved in tsunami research, they contribute freely to the program and will form the vital core of the scientific effort when an event occurs. In addition, we have volunteer professionals such as the ASCE members and observers who range from retired scientists to community college instructors to federal employees. Anyone organizing a monitoring effort should give their first attention to building and maintaining such a team.

#### IV. REHEARSALS AND RESULTS

We have found rehearsals, drills, and tests to be essential to training and properly involving personnel - volunteers and official - and for establishing and improving the procedures. Such activities:

- keep our name and phone lists updated for call-up procedure,

- keep track of changes in official organizations and staff,
- get people to meet and work with each other,
- validate or modify our procedures,
- exercise equipment (especially the cameras) and their operators,
- provide continuity and feedback,
- verify access to sites,
- uncover problems,
- interrupt the "forgetting curve".

We hold one exercise a year, normally during a high surf period from a winter storm to serve as a sort of model and provide some of the problems that might actually be encountered. It is most important to conduct any operation with an open attitude towards errors, omissions, and problems. Problems of any sort can be resolved during or after a drill with a proper critique and follow-up. Often a rehearsal can be combined with an exercise conducted for a related purpose, such as the Civil Defense exercise of [S. O. Wigen and M. M. Ward, 1981]. An update of procedures and changes, however, is needed more than once a year. And, never assume that anyone - especially the agencies with their own "chain of command" - will keep you informed of changes. Turnover and changes are something any program of this sort must periodically deal with. Backups for key people, periodic reviews and an occasional exercise are the best methods for preventing any problems later.

An indication of successful result occurred last winter when

Curtis

a drill was held on the north shore of Oahu. Two ground photographers obtained time-lapsed photos of assigned areas while a Navy P-3 obtained repeated strip photos and a CAP L-19 took oblique photos simultaneously. Timing was set by arrival of storm surf, and both the call-out and observer notification done by standard procedures.

## V. CONCLUSION

Prepare now for an unusual event tomorrow that may not happen for years.

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- Curtis, George D., "Priority for Real-Time Aerial Photography of Tsunami", Joint Inst. for Marine and Atmospheric Research Report, May 1980. (incorporating information from applicable papers of W. Adams, D. Cox and H. Loomis)
- National Science Foundation and National Oceanic and Atmospheric Administration, Tsunami Research Opportunities, Washington D. C., Sept. 1981.
- State of Hawaii, Director of Civil Defense, "EOSE-81 (Tsunami) Scenario", Aug. 1981.
- Wigen, S. O. and M. M. Ward, "Post-Tsunami Disaster Survey", Lighthouse, Journal of Canadian Hydrographers Assn., April 1981.

MEMBERSHIP INFORMATION:      **The Tsunami Society**  
   **Box 8523**  
   **Honolulu, HI 96815, USA**

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**Application for Membership**

**THE TSUNAMI SOCIETY**  
P.O. Box 8523  
Honolulu, Hawaii 96815, USA

I desire admission into the Tsunami Society as: (Check appropriate box.)

Student     Associate     Active     Sustaining     Corporate Member

Name \_\_\_\_\_ Signature \_\_\_\_\_

Address \_\_\_\_\_ Phone No. (    ) \_\_\_\_\_

City \_\_\_\_\_ Date of Birth \_\_\_\_\_

Zip Code \_\_\_\_\_ Country \_\_\_\_\_ Country of Birth \_\_\_\_\_

Employed by \_\_\_\_\_

Address \_\_\_\_\_

Title of your position \_\_\_\_\_

A Student Member Applicant needs the signature of a faculty member verifying the student's status.

Faculty Member \_\_\_\_\_ School \_\_\_\_\_

Applicant should include a resumé covering Professional Record, College Degrees, and Publications. Emphasis of tsunami interest will be helpful. All information submitted will be considered true and available for public dissemination.

Fee: Student: \$5.00 Associate: \$10.00 Active: \$10.00 Sustaining: \$50.00 Corporate: \$100.00

Send dues for one year with application. Membership shall date from 1 January of the year in which the applicant is elected to membership. Membership of an applicant applying on or after October 1 will begin with January of the succeeding calendar year and his first dues payment will be applied to that year.

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City \_\_\_\_\_ Date of Birth \_\_\_\_\_

Zip Code \_\_\_\_\_ Country \_\_\_\_\_ Country of Birth \_\_\_\_\_

Employed by \_\_\_\_\_

Address \_\_\_\_\_

Title of your position \_\_\_\_\_

A Student Member Applicant needs the signature of a faculty member verifying the student's status.

Faculty Member \_\_\_\_\_ School \_\_\_\_\_

Applicant should include a resume' covering Professional Record, College Degrees, and Publications. Emphasis of tsunami interest will be helpful. All information submitted will be considered true and available for public dissemination.

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